SPE Petroleum Engineering Certification and PE License Exam Reference Guide
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Foreword

The Society of Petroleum Engineers (SPE) has a vision to “enable the global oil and gas E&P industry to share technical knowledge needed to meet the world’s energy needs in a safe and environmentally responsible manner.” One way of achieving this vision is by sustaining the competency, professionalism, impartiality, and integrity of the personnel within the industry. SPE has responded to this challenge by establishing the SPE Professional Certification Exam (SPEC), which offers members a vehicle to develop their technical competencies and skills across the entire field of petroleum engineering. The SPEC is internationally recognized and represents a high standard of knowledge in different areas of petroleum engineering via an exam that includes engineering fundamentals and complex practical problems.

The SPEC has been offered internationally now for 13 years and is complemented by an SPE short course that gives candidates insight into the range of topics that the exam will cover and the style of questions that they will face. Initially, no specific course manual existed; in most cases, the SPE Petroleum Engineering Handbook series was the main source of reference for the course.

In the summer of 2008, the SPE Engineering Professionalism Committee discussed the idea of writing a book that could be used as a single reference for the SPEC and exam review course. The initial concept of the book was that of a one-stop, go-to reference for future oil and gas industry professionals, with all major concepts, equations, charts, tables, and formulas between its covers. This soon evolved into a “Quick Reference Guide for Petroleum Engineers,” but because the primary intended use for the book was as a reference for the SPEC and US PE Exam, it finally evolved into the guide you are holding today.

The guide has been written for a wide range of audiences and, therefore, will have many applications and uses. It will be of value to university students, recent graduates, and young professionals within the oil and gas industry and academia. However, it is also largely intended for use by experienced professionals who are working on day-to-day projects and require access to a broader scope of petroleum engineering than what falls immediately within their specific areas of expertise.

Along with being of great value before and during examinations, the guide will also be of great use in the workplace. This guide complements the Petroleum Engineering Handbook series by summarizing all of the concepts in a single volume. As a result, there is no need to carry a suitcase full of books on every assignment. The guide is expected to become commonplace in every department and on every desk, platform, or rig in the oil and gas industry. Additionally, the guide is anticipated to be the first-stop reference when oil industry professionals are faced with any upstream or downstream problem.

With this in mind, the SPE Petroleum Engineering Certification and PE License Exam Reference Guide was written in a way that will allow oil industry professionals to apply a formula or equation (that may not be at the forefront of their minds) to their daily processes and procedures without having to cross-reference other texts. The fact that the guide is largely aimed at professionals who have been in the industry for some time allows the user to be familiar with the concepts behind the procedures, so there is no need for a real textbook-style explanation behind their derivation.

In tune with the SPE vision, daily use of the guide by working engineers will increase professional standards and knowledge sharing, thus creating an industry that “meets the world’s energy needs in a safe and environmentally responsible manner.”

Dr. Mohammed Razik Shaikh, SPEC

The Subcommittee recognizes past and present committee members for their encouragement in preparing this guide. The Subcommittee also recognizes the review effort by the SPE Petroleum Engineering Certification and US Engineering Registration Subcommittees. The author acknowledges the assistance of Mr. Foag Haeri in literature search and gathering of information.

SPE extends its appreciation to the SPE Engineering Registration Committee for their efforts in revising the guide in 2019. Specific thanks go to Samuel Cappo, Jarrod Sparks, Paul Lammers, Chris Chamblee, Mark Fisk, Weldon Ransbarger, Lucas Moore, Neal Howard, Eric Robertson, Farrukh Hamza, George Stutz, Jared Clark, David Gaudin, Steven Tkach, and other volunteer contributors.
Chapter 1

Reservoir Engineering

1.1 Volume Calculations

Original Oil in Place in Volumetric Undersaturated Oil Reservoirs

**Volumetric Method**

*Above Bubble Point Pressure*

\[ N = \frac{7.758 \times A \times h \times \phi \times (1 - S_{wi})}{B_{oi}} \]

- 7,758 Number of barrels per acre-foot, bbl/acre-ft
- \( N \) Original oil in place, STB
- \( A \) Area of the zone, acres
- \( h \) Average net thickness of the zone, ft
- \( \phi \) Porosity, unitless
- \( B_{oi} \) Oil formation volume factor at initial reservoir pressure, bbl/STB
- \( S_{wi} \) Water saturation at initial reservoir conditions, unitless

*Below Bubble Point Pressure*

\[ N = \frac{7.758 \times A \times h \times \phi \times (1 - S_{wi} - S_{w})}{B_{o}} \]

- 7,758 Number of barrels per acre-foot, bbl/acre-ft
- \( N \) Original oil in place, STB
- \( A \) Area of the zone, acres
- \( h \) Average net thickness of the zone, ft
- \( \phi \) Porosity, unitless
- \( B_{o} \) Oil formation volume factor, bbl/STB
**Material Balance Method Without Water Influx**

**Above Bubble Point Pressure**

\[ N = \frac{N_p B_o}{B_o \left( c_o S_o + c_w S_w + c_f \right) \Delta p} \]

- \( N_p \): Original oil in place, STB
- \( N \): Cumulative oil produced, STB
- \( B_o \): Oil formation volume factor, bbl/STB
- \( B_w \): Oil formation volume factor at initial reservoir pressure, bbl/STB
- \( \Delta p \): Change in volumetric reservoir pressure, psi
- \( c_o \): Oil compressibility, psi\(^{-1}\)
- \( c_w \): Water compressibility, psi\(^{-1}\)
- \( c_f \): Formation compressibility, psi\(^{-1}\)
- \( S_o \): Oil saturation, unitless
- \( S_w \): Water saturation at initial reservoir conditions, unitless

**Below Bubble Point Pressure**

\[ N = \frac{N_p \left( B_o + (R_p - R_w) B_g \right)}{B_g - B_o} \]

- \( N_p \): Original oil in place, STB
- \( N \): Cumulative oil produced, STB
- \( B_o \): Oil formation volume factor, bbl/STB
- \( B_w \): Two-phase formation volume factor at initial reservoir pressure, bbl/STB
- \( B_g \): Gas formation volume factor, bbl/scf
- \( R_p \): Cumulative produced gas/oil ratio, scf/STB
- \( R_w \): Solution gas/oil ratio at initial reservoir pressure, scf/STB
- \( R_g \): Solution gas/oil ratio, scf/STB
- \( B_o \): Oil formation volume factor, bbl/STB

**Recovery Factor**

\[ RF = \frac{N_p}{N} = \frac{\left( B_i - B_o \right)}{\left( B_i + (R_p - R_w) B_g \right)} \]

- \( RF \): Recovery factor, fraction
- \( N_p \): Cumulative oil produced, STB
- \( N \): Original oil in place, STB
- \( B_i \): Two-phase formation volume factor, bbl/STB
- \( B_o \): Two-phase formation volume factor at initial reservoir pressure, bbl/STB
- \( B_g \): Gas formation volume factor, bbl/scf
- \( R_p \): Cumulative produced gas/oil ratio, scf/STB
- \( R_w \): Solution gas/oil ratio at initial reservoir pressure, scf/STB
- \( R_g \): Solution gas/oil ratio, scf/STB
- \( B_o \): Oil formation volume factor, bbl/STB
Original Oil in Place in Undersaturated Oil Reservoirs With Water Influx

**Volumetric Method**

\[
N = \frac{7.758 \times A \times h \times \phi \times (1 - S_{wi} - S_{or})}{B_{oi}}
\]

7.758  Number of barrels per acre-foot, bbl/acre-ft
\(N\)  Original oil in place, STB
\(A\)  Area of the zone, acres
\(h\)  Average net thickness of the zone, ft
\(\phi\)  Porosity, unitless
\(B_{oi}\)  Oil formation volume factor at initial reservoir pressure, bbl/STB
\(S_{wi}\)  Water saturation at initial reservoir conditions, unitless
\(S_{or}\)  Residual oil saturation, unitless

**Material Balance Method**

\[
N = \frac{N_p \left[ B_i + (R_p - R_s) B_e \right] - W_e + B_w W_p}{B_t - B_i + B_e \left( c_w S_{wi} + c_f \right) / \Delta \bar{p}}
\]

\(N\)  Initial oil in place, STB
\(N_p\)  Cumulative oil produced, STB
\(B_i\)  Two-phase formation volume factor, bbl/STB = \(B_o + B_g (R_o - R_w)\)
\(B_e\)  Two-phase formation volume factor at initial reservoir pressure, bbl/STB
\(B_w\)  Water formation volume factor, bbl/STB
\(B_g\)  Gas formation volume factor, bbl/scf
\(\Delta \bar{p}\)  Change in reservoir pressure, psi
\(c_w\)  Water compressibility, psi\(^{-1}\)
\(c_f\)  Formation compressibility, psi\(^{-1}\)
\(W_p\)  Cumulative water produced, STB
\(W_e\)  Water influx, bbl
\(R_p\)  Cumulative produced gas/oil ratio, scf/STB
\(R_s\)  Solution gas/oil ratio at initial reservoir pressure, scf/STB
\(S_{wi}\)  Water saturation at initial reservoir conditions, unitless
\(B_g\)  Oil formation volume factor, bbl/STB
\(R_s\)  Solution gas/oil ratio, scf/STB

**Oil Unit Recovery Factor (RF)**

(Water drive with no appreciable decline in reservoir pressure)

\[
RF = \frac{N_p E}{N} = \frac{7.758 \times \phi \times (1 - S_{wi} - S_{or})}{B_{oi}} \text{ STB/acre-ft}
\]

**Oil Recovery Efficiency (RE)**

(Water drive with no appreciable decline in reservoir pressure)

\[
RE = 100 \left( \frac{1 - S_{wi} - S_{or}}{1 - S_{or}} \right) \text{ %}
\]

\(N\)  Original oil in place, STB
\(\phi\)  Porosity, unitless
\(B_{oi}\)  Oil formation volume factor at initial reservoir pressure, bbl/STB
**Original Oil in Place in Saturated Oil Reservoirs**

**Volumetric Method**

\[
N = \frac{7,758 \times A_{oi} \times h_{oi} \times \phi_{oi} \times (1 - S_{wioz})}{B_{oi}}
\]

7,758 \hspace{1em} \text{Number of barrels per acre-foot, bbl/acre-ft}

\(N\) \hspace{1em} \text{Original oil in place, STB}

\(A_{oi}\) \hspace{1em} \text{Area of the oil zone, acres}

\(h_{oi}\) \hspace{1em} \text{Average net thickness of the oil zone, ft}

\(\phi_{oi}\) \hspace{1em} \text{Average porosity in the oil zone, unitless}

\(B_{oi}\) \hspace{1em} \text{Oil formation volume factor at initial reservoir pressure, bbl/STB}

\(S_{wioz}\) \hspace{1em} \text{Initial average connate water saturation in the oil zone, unitless}

**Material Balance Method: (With Water Influx)**

\[
N = \frac{N_{p} \left[ B_{o} \left( R_{p} - R_{w} \right) - B_{w} \right]}{B_{oi}} + m B_{oi} \left( B_{oi} - B_{w} \right)
\]

\(m = \frac{GB_{oi}}{NB_{oi}}\)

\(N\) \hspace{1em} \text{Initial oil in place, STB}

\(N_{p}\) \hspace{1em} \text{Cumulative oil produced, STB}

\(B_{o}\) \hspace{1em} \text{Oil formation volume factor, bbl/STB}

\(B_{oi}\) \hspace{1em} \text{Oil formation volume factor at initial reservoir pressure, bbl/STB}

\(B_{t}\) \hspace{1em} \text{Two-phase formation volume factor, bbl/STB} = B_{o} + B_{g} \left( R_{po} - R_{wo} \right)

\(B_{g}\) \hspace{1em} \text{Gas formation volume factor, bbl/scf}

\(B_{gi}\) \hspace{1em} \text{Gas formation volume factor at initial reservoir pressure, bbl/scf}

\(W_{p}\) \hspace{1em} \text{Cumulative water produced, STB}

\(W_{w}\) \hspace{1em} \text{Water influx, bbl}

\(R_{p}\) \hspace{1em} \text{Cumulative produced gas/oil ratio, scf/STB}

\(R_{so}\) \hspace{1em} \text{Solution gas/oil ratio, scf/STB}

\(R_{s}\) \hspace{1em} \text{Solution gas/oil ratio at initial reservoir pressure, scf/STB}

\(m\) \hspace{1em} \text{Ratio of initial reservoir free gas volume to initial reservoir oil volume, unitless}

\(G\) \hspace{1em} \text{Original gas in place, scf}

**Original Gas in Place in Volumetric Dry Gas, Wet Gas, and Retrograde Gas Condensate Reservoirs**

**Volumetric Method**

\[
G = \frac{7,758 \times A \times h \times \phi \times (1 - S_{w})}{B_{gi}}
\]
Alternatively, if $B_{gi}$ is defined as cf/scf where cf is ft$^3$,

$$G = \frac{43,560 \times A \times h \times \phi \times (1 - S_{wi})}{B_{gi}}$$

7,758 Number of barrels per acre-foot, bbl/acre-ft
43,560 Number of cubic feet per acre-foot, cf/acre-ft
$G$ Original gas in place, Mscf (or scf if alternative equation with $B_{gi}$ as cf/scf)
$A$ Area of the zone, acres
$h$ Average net thickness of the zone, ft
$\phi$ Porosity, unitless
$B_{gi}$ Gas formation volume factor at initial reservoir pressure, bbl/Mscf (or cf/scf for alternative equation)
$S_{wi}$ Water saturation at initial reservoir conditions, unitless

**Applying Bulk Reservoir Volume ($V_b$)**

$$G = \frac{43,560 \times V_b \times \phi \times (1 - S_{wi})}{B_{gi}}$$

**Applying Reservoir Pore Volume ($V_p$)**

$$G = \frac{43,560 \times V_p \times (1 - S_{wi})}{B_{gi}}$$

$G$ Original gas in place, scf
$V_b$ Bulk reservoir volume, acre-ft $= A \times h$
$V_p$ Reservoir pore volume, acre-ft $= A \times h \times \phi = V_b \times \phi$
$\phi$ Porosity, unitless
$S_{wi}$ Water saturation at initial reservoir conditions, unitless
$B_{gi}$ Gas formation volume factor at initial reservoir pressure, cf/scf

**Material Balance Method**

$$\frac{p}{z} = \frac{p_i}{z_i} \left(1 - \frac{G_p}{G}\right)$$

**Recovery Factor Using “B” Terms**

$$RF = \frac{G_p}{G} = \frac{B_p - B_{gi}}{B_{gi}}$$

**Recovery Factor Using “p/z” Terms**

$$RF = \frac{G_p}{G} = \frac{\left(\frac{p_i}{z_i} - \frac{p}{z}\right)}{\frac{p_i}{z_i}}$$

$RF$ Recovery factor, fraction
$G$ Original gas in place, Mscf
$G_p$ Cumulative gas produced, Mscf
$p$ Reservoir pressure (current or abandonment conditions), psia
$p_i$ Initial reservoir pressure, psia
$z$ Gas compressibility factor (current or abandonment conditions), unitless
Gas compressibility factor at initial reservoir pressure, unitless

Gas formation volume factor at initial reservoir pressure, bbl/Mscf

Gas formation volume factor (current or abandonment conditions), bbl/Mscf

**Gas Formation Volume Factor**

\[ B_g = \frac{p_z z T}{T_p p} \]

\[ B_g = \frac{0.00504 z T}{p} = \text{[=]} \text{ bbl / scf} \]

Alternatively, if \( B_g \) is defined as cf/scf where cf is ft³,

\[ B_g = \frac{0.02829 z T}{p} = \text{[=]} \text{ cf / scf} \]

\( B_g \) Gas formation volume factor, bbl/scf (or cf/scf for alternative equation)

\( p_z \) Pressure at standard conditions, or pressure base, psia = 14.7 psia

\( T_p \) Temperature at standard conditions, °R (°R = °F + 460) = 60°F = 520°R

\( z \) Gas compressibility factor or gas deviation factor, unitless

\( T \) Reservoir temperature, °R

\( p \) Reservoir pressure, psia

**Pseudoreduced Pressure and Temperature**

\[ p_{pr} = \frac{p}{p_{pc}} \]

\[ T_{pr} = \frac{T}{T_{pc}} \]

\( p_{pr} \) Pseudoreduced pressure, unitless

\( p \) Pressure, psia

\( p_{pc} \) Pseudocritical pressure, psia

\( T_{pr} \) Pseudoreduced temperature, unitless

\( T \) Temperature, °R (°R = °F + 460)

\( T_{pc} \) Pseudocritical temperature, °R

**Specific Gravity of a Gas**

\[ \gamma_g = \frac{\rho_g}{\rho_{air}} = \frac{M_g}{M_{air}} = \frac{M_g}{28.97} \]  

(assumes gas and air obey the ideal-gas law)

also \( \gamma_g = \frac{M_g}{28.97} \)

\[ M_a = \sum_j \gamma_j M_j \]

\( \gamma_g \) Specific gravity of a gas, unitless

\( \rho_g \) Density of a gas, lbm/ft³

\( \rho_{air} \) Density of air, lbm/ft³

\( M_g \) Molecular weight of a gas, lbm/lbm-mol

\( M_{air} \) Molecular weight of air (= 28.97 lbm/lbm-mol)

\( M_a \) Apparent molecular weight of a gas mixture, lbm/lbm-mol
γ<sub>j</sub>  Mole fraction of gas component j in a gas mixture, fraction

M<sub>j</sub>  Molecular weight of gas component j in a gas mixture, lbm/lbm-mol

**Specific Gravity of a Reservoir Gas for a One-Stage Separation System**

\[ \gamma_o = \frac{R_i \gamma + 4,602 \gamma_o}{R_i + 133,316 \gamma / M_o} \]

\[ M_o = \frac{5,954}{\gamma_{API} - 8.811} \]

\[ M_o = \frac{42.43 \gamma_o}{1.008} \]

\[ \gamma_o = \frac{141.5}{\gamma_{API} + 131.5} \]

γ<sub>o</sub>  Specific gravity of reservoir gas, unitless

R<sub>i</sub>  Primary (high pressure) separator gas to stock-tank liquid ratio, scf/STB

γ<sub>i</sub>  Specific gravity of primary separator gas, unitless (air = 1.0)

γ<sub>API</sub>  Specific gravity of stock-tank hydrocarbon liquid in °API

γ<sub>o</sub>  Specific gravity of the liquid hydrocarbons, unitless (water = 1.0)

M<sub>o</sub>  Molecular weight of stock-tank liquid (condensate), lbm/lbm-mol

**Specific Gravity of a Reservoir Gas for a Three-Stage Separation System**

\[ \gamma_o = \frac{R_i \gamma + 4,602 \gamma_o + R_i \gamma + R_2 \gamma_2}{R_i + (133,316 \gamma / M_o) + R_2 + R_3} \]

γ<sub>2</sub>  Specific gravity of secondary separator gas, unitless

γ<sub>3</sub>  Specific gravity of stock-tank gas, unitless

R<sub>2</sub>  Secondary (low pressure) separator gas to stock-tank liquid ratio, scf/STB

R<sub>3</sub>  Stock-tank gas to stock-tank liquid ratio, scf/STB

**Original Gas in Place in Gas Reservoirs With Water Influx**

**Material Balance Method**

\[ G = \frac{G_p B_s - W_p + B_g W_p}{B_s - B_{gi}} \]

G  Initial gas in place, Mcf

G<sub>p</sub>  Cumulative gas produced, Mcf

B<sub>s</sub>  Water formation volume factor, bbl/STB

B<sub>g</sub>  Gas formation volume factor, bbl/Mcf

B<sub>gi</sub>  Gas formation volume factor at initial reservoir pressure, bbl/Mcf

W<sub>p</sub>  Cumulative water produced, STB

W<sub>i</sub>  Water influx, bbl

**Gas Unit Recovery Factor (RF)**

\[ RF = 43,560 \times \phi \left( \frac{1-S_{wi}}{B_g} - \frac{S_{gr}}{B_{gi}} \right) \text{ scf / acre-ft} \]
**Gas Recovery Efficiency (RE)**

\[
RE = \frac{100 \left( 1 - \frac{S_{wi}}{B_{gi}} \right)}{\left( 1 - \frac{S_{gr}}{B_{ga}} \right)} [\%]
\]

- **RF** Gas unit recovery factor, scf/acre-ft
- **RE** Gas recovery efficiency, %
- **φ** Porosity, unitless
- **S_{wi}** Water saturation at initial reservoir conditions, unitless
- **S_{gr}** Residual gas saturation, unitless
- **B_{gi}** Gas formation volume factor at initial reservoir pressure, bbl/scf or cf/scf
- **B_{ga}** Gas formation volume factor at abandonment reservoir pressure, bbl/scf or cf/scf

**Material Balance Expressed as a Linear Equation (by Havlena and Odeh 1963)**

\[
F = NE_o + W_o B_o
\]

\[
F = N \left( E_o + m E_g + E_{f,w} \right) + W_o B_o
\]

\[
F = N_p \left[ B_o \left( R_p - R_{oi} \right) B_{gi} \right] + W_p B_o
\]

\[
E_o = (B_o - B_{wi}) + \left( R_{oi} - R_{wi} \right) B_{gi}
\]

\[
E_g = B_{wi} \left( \frac{B_{gi}}{B_{ji}} - 1 \right)
\]

\[
E_{f,w} = (1 + m) B_{wi} \left( \frac{c_w S_{wi} + c_f}{1 - S_{wi}} \right) \Delta p
\]

\[
E_t = E_o + m E_g + E_{f,w}
\]

- **F** Underground withdrawal
- **E_o** Oil and solution gas expansion
- **E_g** Gas cap expansion
- **E_{f,w}** Hydrocarbon space reduction
- **E_t** Total expansion
- **N** Initial oil in place, STB
- **N_p** Cumulative oil produced, STB
- **B_o** Oil formation volume factor, bbl/STB
- **B_g** Gas formation volume factor, bbl/scf
- **B_{wi}** Oil formation volume factor at initial reservoir pressure, bbl/STB
- **B_{gi}** Gas formation volume factor at initial reservoir pressure, bbl/scf
- **Δp** Change in reservoir pressure, psi
- **c_w** Water compressibility, psi\(^{-1}\)
- **c_f** Formation compressibility, psi\(^{-1}\)
- **S_{wi}** Water saturation at initial reservoir conditions, unitless
- **R_p** Cumulative produced gas/oil ratio, scf/STB
- **R_{oi}** Initial solution gas/oil ratio, scf/STB
- **R_{wi}** Solution gas/oil ratio, scf/STB
- **m** Ratio of initial reservoir free gas volume to initial reservoir oil volume, unitless
Formation Compressibility (Newman Correlations)

**Isothermal Compressibility**

\[ c = -\frac{1}{V_o} \left( \frac{dV_o}{dp} \right) \]

**Consolidated Sandstones Under Hydrostatic Pressure**

\[ c_f = \frac{97.3200(10)^{-6}}{1+55.8721\phi^{1.42859}} \] (where \(0.02 < \phi < 0.23, c_f \pm 2.6\%\))

**Limestone Formations Under Hydrostatic Pressure**

\[ c_f = \frac{0.853531}{1+2.47664(10)^6 \phi^{0.92990}} \] (where \(0.02 < \phi < 0.33, c_f \pm 11.6\%\))

**Reservoir Bulk Volume**

**Simpson’s Rule (valid for odd-numbered layers only)**

\[ n = \text{odd only (most accurate method)} \]

\[ V_b = \frac{h}{3} (A_1 + 4A_2 + 2A_3 + 4A_4 + 2A_5 + \ldots + 4A_{n-1} + A_n) + \frac{h}{3} A_n \]

**Trapezoidal Rule (valid for all numbered layers)**

\[ n = \text{even or odd (upper area at least 1/2 of lower)} \]

\[ V_b = \frac{h}{2} (A_1 + 2A_2 + 2A_3 + 2A_4 + \ldots + 2A_{n-1} + A_n) + \frac{h_n}{2} A_n \]

**Pyramid Rule (valid for each layer only)**

Note: For the Pyramid Rule, \(A_n\) is the area enclosed by the lower isopach line and \(A_{n+1}\) is the area enclosed by the upper isopach line.

\[ \Delta V_b = \frac{h}{3} \left( A_n + A_{n+1} + \sqrt{A_n A_{n+1}} \right) \]

\(V_b\) Reservoir bulk volume, acre-ft

\(\Delta V_b\) Bulk volume for single or individual layer, acre-ft

\(h\) Interval between isopach lines or common height of layers excluding top layer, ft (also called contour interval)

\(h_n\) Height of top layer, ft

\(n\) Number of layers, unitless

\(A_i\) Area of bottom-most interval, acres
A_2 \quad \text{Area of second-from-bottom interval, acres}
A_{3,4,5} \quad \text{Area of designated interval counting from bottom, acres}
A_{n-1} \quad \text{Area of second-to-last or second-from-top interval, acres}
A_n \quad \text{Area of top-most interval, acres}

1.2 Drive Mechanisms

Depletion Drive Index (DDI) or Solution-Gas Drive Index (SGDI)

\[
\text{DDI} = \frac{N (B_o - B_n)}{N_p \left[ B_t + (R_o - R_{soi}) B_g \right]}
\]

Segregation (Gas Cap) Drive Index (SDI) or Gas Cap Drive Index (GCDI)

\[
\text{SDI} = \frac{G (B_e - B_{so})}{N_p \left[ B_t + (R_o - R_{so}) B_g \right]}
\]

Water Drive Index (WDI)

\[
\text{WDI} = \frac{W_e - W_o B_n}{N_p \left[ B_t + (R_o - R_{so}) B_g \right]}
\]

Expansion Drive Index (EDI), also referred to as Formation and Connate Water Compressibility Index (CDI) or Pore Volume Contraction Index (PVCI)

\[
\text{EDI} = \frac{NB_{soi} \left(1 + m \frac{c_w S_{wi} + c_f}{1 - S_{wi}} \right) \Delta p}{N_p \left[ B_t + (R_o - R_{so}) B_g \right]}
\]

\[
\text{DDI} + \text{SDI} + \text{WDI} + \text{EDI} = 1
\]

N \quad \text{Initial oil in place, STB}
N_p \quad \text{Cumulative oil produced, STB}
G \quad \text{Initial gas in place, scf}
W_e \quad \text{Water influx into reservoir, bbl}
W_p \quad \text{Cumulative water produced, STB}
B_t \quad \text{Two-phase formation volume factor, bbl/STB, } B_t = B_o + B_g (R_{so} - R_{soi})
B_o \quad \text{Oil formation volume factor, bbl/STB}
R_o \quad \text{Cumulative produced gas/oil ratio, scf/STB}
R_{soi} \quad \text{Initial solution gas/oil ratio, scf/STB}
R_{so} \quad \text{Solution gas/oil ratio, scf/STB}
B_{soi} \quad \text{Initial two-phase formation volume factor, bbl/STB, } B_{soi} = B_{so}
B_{oi} \quad \text{Initial oil formation volume factor, bbl/STB}
B_g \quad \text{Gas formation volume factor, bbl/scf}
B_{gi} \quad \text{Initial gas formation volume factor, bbl/scf}
B_w \quad \text{Water formation volume factor, bbl/STB}
c_w \quad \text{Water compressibility, psi}^{-1}
c_f \quad \text{Formation compressibility, psi}^{-1}
S_{wi} \quad \text{Water saturation at initial reservoir conditions, unitless}
\Delta p \quad \text{Change in reservoir pressure, psi}
m \quad \text{Ratio of initial reservoir free gas volume to initial reservoir oil volume, unitless}
Solution Gas Drive Mechanism

Above Bubble Point Pressure

\[
N_p B_o = NB_o \left[ c_o + \left( \frac{c_o S_{wi} + c_f}{1 - S_{wi}} \right) \Delta p \right]
\]

\[
N = \frac{N_p B_o}{B_{wi} \left[ \frac{(B_o - B_w)}{B_{wi}} + \left( \frac{c_o S_{wi} + c_f}{1 - S_{wi}} \right) \Delta p \right]}
\]

Below Bubble Point Pressure

\[
N_p \left[ B_o + \left( R_p - R_m \right) B_t \right] = N \left[ \left( B_o - B_w \right) + \left( R_m - R_o \right) B_t \right]
\]

\[
N = \frac{N_p \left[ B_o + \left( R_p - R_m \right) B_t \right]}{\left( B_o - B_w \right) + \left( R_m - R_o \right) B_t}
\]

Gas-Cap Drive Mechanism

\[
N_p \left[ B_o + \left( R_p - R_m \right) B_t \right] = NB_{wi} \left[ \frac{(B_o - B_w) + (R_m - R_o) B_t}{B_{wi}} + m \left( \frac{B}{B_{gf}} - 1 \right) \right]
\]

\[
N = \frac{N_p \left[ B_o + \left( R_m - R_o \right) B_t \right]}{B_{wi} \left[ \frac{(B_o - B_w) + (R_m - R_o) B_t}{B_{wi}} + m \left( \frac{B}{B_{gf}} - 1 \right) \right]}
\]

Water Drive Mechanism

\[
W_o = \left( c_o + c_f \right) W_p \Delta p
\]

\[ N \] Initial oil in place, STB
\[ N_p \] Cumulative oil produced, STB
\[ B_o \] Oil formation volume factor, bbl/STB
\[ B_t \] Gas formation volume factor, bbl/scf
\[ B_{wi} \] Initial oil formation volume factor, bbl/STB
\[ B_{wi} \] Initial gas formation volume factor, bbl/scf
\[ \Delta p \] Change in reservoir pressure, psi
\[ c_o \] Oil compressibility, psi\(^{-1}\)
\[ c_w \] Water compressibility, psi\(^{-1}\)
\[ c_f \] Formation compressibility, psi\(^{-1}\)
\[ S_{wi} \] Water saturation at initial reservoir conditions, unitless
\[ R_p \] Cumulative produced gas/oil ratio, scf/STB
\[ R_m \] Initial solution gas/oil ratio, scf/STB
\[ R_{soi} \] Solution gas/oil ratio, scf/STB
\[ W_o \] Water influx into reservoir, bbl
\[ W_i \] Initial volume of water, bbl
\[ m \] Ratio of initial reservoir free gas volume to initial reservoir oil volume, unitless
1.3 Stages of Production

Darcy’s Law

\[ u = -\frac{k}{\mu} \frac{dp}{dL} \]

- \( u \): Velocity, ft/sec
- \( k \): Permeability, md
- \( \mu \): Viscosity, cp
- \( p \): Pressure, psia
- \( L \): Length, ft

Steady-State Linear Flow of Incompressible Fluids

\[ q = \frac{0.001127 \ k \ A \ (p_1 - p_2)}{\mu \ L} \]

- \( q \): Flow rate, B/D
- \( k \): Absolute permeability, md
- \( p \): Pressure, psia
- \( \mu \): Viscosity, cp
- \( L \): Length, ft
- \( A \): Cross-sectional area of flow, ft²
- \( p_1 \): Upstream pressure, psia
- \( p_2 \): Downstream pressure, psia

Steady-State Linear Flow of Slightly Compressible Fluids

\[ q_{ref} = \left( \frac{0.001127 \ k \ A}{\mu \ c \ L} \right) \ln \left[ \frac{1 + c(p_{ref} - p_2)}{1 + c(p_{ref} - p_1)} \right] \]

- \( q_{ref} \): Flow rate at a reference pressure \( p_{ref} \), B/D
- \( p_1 \): Upstream pressure, psia
- \( p_2 \): Downstream pressure, psia
- \( k \): Absolute permeability, md
- \( \mu \): Viscosity, cp
- \( c \): Average liquid compressibility, psi⁻¹
- \( L \): Length, ft
- \( A \): Cross-sectional area of flow, ft²
- \( p_{ref} \): Reference pressure, psia = 14.7 psia at standard conditions

Steady-State Linear Flow of Compressible Fluids

\[ Q_{sc} = \frac{0.003164 \ T_w \ A \ k \ (p_1^2 - p_2^2)}{p_w \ T \ L \ z \ \mu_g} \]

- \( Q_{sc} \): Gas flow rate at standard conditions, Mscf/D
- \( p_1 \): Upstream pressure, psia
- \( p_2 \): Downstream pressure, psia
- \( k \): Permeability, md
- \( T \): Reservoir temperature, °R
- \( T_w \): Temperature at standard conditions, °R (°R = °F + 460), 60°F = 520°R
- \( \mu_g \): Gas viscosity, cp
- \( A \): Cross-sectional area of flow, ft²
**Steady-State Radial Flow of Incompressible Fluids**

\[ Q_o = \frac{0.00708 k h (p_e - p_{wf})}{\mu_o B_o \ln(r_e / r_w) + s} \]

- \( Q_o \): Oil flow rate, STB/D
- \( p_e \): External pressure, psia
- \( p_{wf} \): Bottomhole flowing pressure, psia
- \( k \): Permeability, md
- \( \mu_o \): Oil viscosity, cp
- \( B_o \): Oil formation volume factor, bbl/STB
- \( h \): Average net thickness of the zone, ft
- \( r_e \): External or drainage radius, ft
- \( r_w \): Effective wellbore radius, ft
- \( s \): Skin factor, unitless

**Steady-State Radial Flow of Slightly Compressible Fluids**

\[ q_{ref} = \frac{0.00708 kh}{\mu c \ln(r_e / r_w)} \ln \left[ \frac{1 + c_o (14.7 - p_e)}{1 + c (p_{ref} - p_e)} \right] \]

\[ Q_o = \frac{0.00708 kh}{\mu_o B_o c_o \ln(r_e / r_w) + s} \ln \left[ \frac{1 + c_o (14.7 - p_e)}{1 + c (p_{ref} - p_e)} \right] \]

- \( q_{ref} \): Flow rate at a reference pressure \( p_{ref} \), B/D
- \( Q_o \): Oil flow rate, STB/D
- \( p_{ref} \): Reference pressure, psia = 14.7 psia at standard conditions
- \( 14.7 \): Pressure at standard conditions, psia, or pressure base = 14.7 psia
- \( p_e \): External pressure, psia
- \( p_{wf} \): Bottomhole flowing pressure, psia
- \( k \): Permeability, md
- \( \mu \): Viscosity, cp
- \( \mu_o \): Oil viscosity, cp
- \( B_o \): Oil formation volume factor, bbl/STB
- \( h \): Average net thickness of the zone, ft
- \( r_e \): External or drainage radius, ft
- \( r_w \): Effective wellbore radius, ft
- \( c \): Average liquid compressibility, psi\(^{-1}\)
- \( c_o \): Oil compressibility coefficient, psi\(^{-1}\)
- \( s \): Skin factor, unitless

**Steady-State Radial Flow of Compressible Fluids**

\[ Q_g = \frac{k h (\psi_e - \psi_{ref})}{1,422 T \ln(r_e / r_w)} \]

\[ Q_{g,(p>2000)} = \frac{k h (p_e^2 - p_{ref}^2)}{1,422 T (\mu_o z)_{avg} \ln(r_e / r_w)} \]

- \( Q_g \): Gas flow rate, Mscf/D
Unsteady-State (Transient) Radial Flow of Slightly Compressible Fluids (Diffusivity Equation)

\[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} = \frac{\phi \mu c_i}{0.006328 k} \frac{\partial p}{\partial t} \]

- \( k \): Permeability, md
- \( h \): Average net thickness of the zone, ft
- \( T \): Reservoir temperature, °R
- \( r_e \): External or drainage radius, ft
- \( r_w \): Effective wellbore radius, ft
- \( \mu_g \): Gas viscosity, cp
- \( z \): Gas compressibility factor, unitless
- \( \psi_e \): Real-gas pseudopressure as evaluated from 0 to \( p_e \), psi²/cp
- \( \psi_w \): Real-gas pseudopressure as evaluated from 0 to \( p_{wf} \), psi²/cp
- \( p_e \): External pressure, psia
- \( p_{wf} \): Bottomhole flowing pressure, psia

Unsteady-State (Transient) Radial Flow of Compressible Fluids (Diffusivity Equation)

\[ \frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r} \frac{\partial m(p)}{\partial r} = \frac{\phi \mu c_i}{0.000264 k} \frac{\partial m(p)}{\partial t} \]

\[ m(p_e) = m(p_i) - 57,895.3 \left( \frac{p_i}{T_i} \right) \left( \frac{Q_i}{k h} \right) \left[ \log \left( \frac{kt}{\phi \mu c_i r_w^2} \right) - 3.23 \right] \]

- \( m(p) \): Real-gas pseudopressure, psi²/cp
- \( p_i \): Initial reservoir pressure, psia
- \( p_{wf} \): Bottomhole flowing pressure, psia
- \( Q_i \): Gas flow rate, Mscf/D
- \( t \): Time, hr
- \( k \): Permeability, md
- \( p_w \): Pressure at standard conditions, or pressure base, psia = 14.7 psia
- \( T_w \): Temperature at standard conditions, °R (°R = °F + 460), 60°F = 520°R
- \( T \): Reservoir temperature, °R
- \( r_w \): Effective wellbore radius, ft
- \( h \): Average net thickness of the zone, ft
- \( \mu_i \): Gas viscosity at the initial pressure, cp
- \( c_i \): Total compressibility coefficient at \( p_i \), psi⁻¹
- \( \phi \): Porosity, unitless
Pseudosteady-State Radial Flow of Slightly Compressible Fluids

\[
Q = \frac{0.00708 \ k \ h \ (\bar{p}_r - p_{wf})}{\mu B \ln \left(\frac{r_i}{r_e}\right) - 0.75 + s}
\]

\[
J = \frac{Q}{\bar{p}_r - p_{wf}} = \frac{kh}{141.2 B \mu \ln \left(\frac{r_i}{r_e}\right) - 0.75 + s}
\]

- \(Q\) Flow rate, STB/D
- \(\bar{p}_r\) Average reservoir pressure, psia, \(=\sqrt{(p_i^2 + p_{wf}^2)/2}\) for \(p < 2,000\)
- \(p_{wf}\) Bottomhole flowing pressure, psia
- \(k\) Permeability, md
- \(h\) Average net thickness of the zone, ft
- \(\mu\) Oil viscosity, cp
- \(B\) Formation volume factor, bbl/STB
- \(r_e\) External or drainage radius, ft
- \(r_w\) Effective wellbore radius, ft
- \(s\) Skin factor, unitless
- \(J\) Productivity index, STB/D/psi
- \(p_r\) External or reservoir pressure, psia

Pseudosteady-State Radial Flow of Compressible Fluids

\[
Q_g = \frac{k h (\bar{p}_r^2 - p_{wf}^2)}{152.2 T \bar{\mu} \ln \left(\frac{r_i}{r_e}\right) - 0.75 + s}
\]

- \(Q_g\) Gas flow rate, Mscf/D
- \(\bar{p}_r\) Average reservoir pressure, psia
- \(p_{wf}\) Bottomhole flowing pressure, psia
- \(k\) Permeability, md
- \(h\) Average net thickness of the zone, ft
- \(T\) Reservoir temperature, °R
- \(\bar{\mu}\) Average gas viscosity, cp
- \(\bar{\zeta}\) Average gas compressibility factor, unitless
- \(r_e\) External or drainage radius, ft
- \(r_w\) Effective wellbore radius, ft
- \(s\) Skin factor, unitless

The \(E_i\) Function Solution to Diffusivity Equation (Constant Rate)

\[
p_{wf} = p_i - \frac{162.6 Q_o B_o \mu_o}{k h} \left[ \log \left( \frac{kt}{\phi \mu_o c_r r_w^2} \right) - 3.23 + 0.87 s \right]
\]

- \(p_{wf}\) Bottomhole flowing pressure, psia
- \(p_i\) Initial reservoir pressure, psia
- \(k\) Permeability, md
- \(t\) Time, hr \(t > 9.48 \times 10^4 (\phi \mu_o c_r^2 / k)\)
- \(h\) Average net thickness of the zone, ft
- \(\mu_o\) Oil viscosity, cp
- \(B_o\) Oil formation volume factor, bbl/STB
- \(Q_o\) Oil flow rate, STB/D
- \(r_w\) Effective wellbore radius, ft
- \(c_i\) Total compressibility, psi\(^{-1}\) \(c_i = c_e S_e + c_n S_n + c_s S_s + c_f\)
- \(\phi\) Porosity, unitless
- \(s\) Skin factor, unitless
1.4 Well Performance

Pressure Drawdown Analysis (or Constant Terminal Rate Solutions)

\[ k = \frac{162.6q\mu B}{mh} \]

\[ s = 1.15\left( \frac{p_i - p_{bas}}{m \phi \mu c r_e^2} - \log \frac{k}{\phi \mu c r_e^2} + 3.23 \right) \]

\[ t_{wbs} = \frac{(200,000 + 12,000s) C_i}{(kh/\mu)} \]

\[ C_i = \frac{qB}{24} \frac{\Delta t}{\Delta p} \]

\( \Delta t \) and \( \Delta p \) are values read from a point on the unit-slope line on log-log plot.

Less acceptable alternative is to use the actual mechanical properties of the well:

**For a Well With a Rising Liquid/Gas Interface in the Wellbore**

\[ C_i = 25.65 \frac{A_{wb}}{\rho} \]

**For a Wellbore Containing Only Single-Phase Fluid (Liquid or Gas)**

\[ C_i = c_{wb} V_{wb} \]

**Reservoir Pore Volume**

\[ V_p = -0.234qB c_i \left( \frac{\partial p_{of}}{\partial t} \right), \text{ ft}^3 \]

\( \left( \frac{\partial p_{of}}{\partial t} \right) \) is the slope of the straight line \( p_{of} \) vs. \( t \) plot on Cartesian graph paper.

**Transient Period**

\[ E_i \text{ form (at any } r): \]

\[ p(r,t) = p_i + \frac{70.6qB \mu}{kh} E_i \left( -\frac{948\phi \mu c r^2}{kt} \right) \]

Log form valid for \( \frac{948\phi \mu c r^2}{kt} < 0.01: \]

\[ p(r,t) = p_i - m \left[ \log \left( \frac{kt}{\phi \mu c r^2} \right) - 3.23 \right] \]
Horner’s Approximation

\[ t_p = 24 \times \frac{N}{q_{last}} \]

\[ p_i - p = -\frac{70.6\mu q_{start} B}{kh} E_1 \left( -\frac{948\phi \mu c_i r_i^2}{kt_p} \right) \]

Pseudosteady State

\[ p_{wf}(t) = p_i - m \left[ \log \left( \frac{4A}{\gamma C_A r_w^2} \right) - 0.87s \right] - \frac{0.2339q_B t}{c_A h \phi} \]

Time (hours) When Pseudosteady-State Begins:

\[ t = \frac{\phi \mu c_i A t_{DA}}{0.000264 k} \]

\[ t_{DA} \] Dependent on reservoir shape factor \( C_A \), which could be read from the table, “Stabilized conditions for \( \frac{kt}{\phi \mu c_i ^2} \) >” column (Fig. 1.7).

Pressure Buildup Analysis

Horner Equation

\[ p_{wf}(\Delta t) = p_i - m \log \left( \frac{t_p + \Delta t}{\Delta t} \right) \]

\[ k = \frac{162.6\mu B}{mh} \]

\[ r_i = \left( \frac{kt}{948\phi \mu c_i} \right)^{\frac{1}{2}} \]

\[ s = 1.151 \left[ (p_{lw} - p_{w,\Delta t=0}) - \log \left( \frac{k}{\mu c_i \phi r_w^2} \right) + 3.23 \right] \]

\[ t_{sw} = \frac{170,000 C e^{0.144 \Delta t}}{(kh / \mu)} \]

\[ C_s = \frac{q_B \Delta t}{24 \Delta p} \]

\( \Delta t \) and \( \Delta p \) are values read from a point on the unit-slope line on log-log plot. Less acceptable alternative is to use the actual mechanical properties of the well:

For a Well With a Rising Liquid/Gas Interface in the Wellbore

\[ C_s = 25.65 \frac{A_{wb}}{\rho} \]

For a Wellbore Containing Only Single-Phase Fluid (Liquid or Gas)

\[ C_s = c_{wb} V_{wb} \]
Interference Testing

\[ r_D = \frac{r}{r_w} \]

\[ t_D = \frac{0.000264 kt}{\phi c_r r_w^2} \]

\[ p_D = \frac{1}{2} \ln \left( \frac{t_D}{t_r^2} \right) + 0.80907 \]

\[ \Delta p = p_i - p_r = \left( \frac{141.2 q B \mu}{kh} \right) p_D \]

- \( p \) Initial reservoir pressure, psia
- \( V_p \) Reservoir pore volume, ft\(^3\)
- \( p_{wf} \) Wellbore flowing pressure, psia
- \( p_w \) Wellbore pressure during buildup or shut-in, psia
- \( p_{i\Delta t=0} \) Wellbore pressure at instant of shut-in, psia
- \( p_{1hr} \) Horner shut-in pressure, psia, at \( \Delta t = 1 \) hr
- \( t \) Elapsed time, hr
- \( t_{DA} \) Dimensionless time based on drainage area \( A \), unitless
- \( t_p \) Effective producing time or pseudo-producing time, hr
- \( \phi \) Porosity, unitless
- \( h \) Average net thickness of the zone, ft
- \( \mu \) Viscosity, cp
- \( B \) Formation volume factor, RB/STB
- \( k \) Permeability, md
- \( c_i \) Total compressibility, psi\(^{-1}\), \( c_i = c_o S_o + c_w S_w + c_g S_g + c_f \)
- \( c_o \) Oil compressibility, psi\(^{-1}\)
- \( c_w \) Water compressibility, psi\(^{-1}\)
- \( c_g \) Gas compressibility, psi\(^{-1}\)
- \( c_f \) Formation compressibility, psi\(^{-1}\)
- \( S_o \) Oil saturation, unitless
- \( S_w \) Water saturation, unitless
- \( S_g \) Gas saturation, unitless
- \( r \) Radius or distance from center of active well to center of observation well, ft
- \( r_D \) Dimensionless radius, unitless
- \( r_i \) Radius of investigation, ft
- \( r_w \) Effective wellbore radius, ft
- \( \Delta p \) Pressure drawdown at the observation well, psi, \( = p_i - p_r \)
- \( \Delta t \) Shut-in time, hr
- \( q \) Flow rate, STB/D
- \( \gamma \) Euler’s constant = 1.781
- \( A \) Drainage area, ft\(^2\)
- \( C_A \) Dietz shape factor, unitless = 31.6 for a circular reservoir
- \( m \) Absolute value of slope of middle-time line, psi/log cycle = \( \frac{162.6 q B \mu}{kh} \)
- \( s \) Skin factor, unitless
- \( C_s \) Wellbore storage constant, bbl/psi
- \( A_{wb} \) Wellbore area, ft\(^2\)
- \( \rho \) Density of liquid in wellbore, lbm/ft\(^3\)
- \( c_{wb} \) Compressibility of liquid in wellbore, psi\(^{-1}\)
- \( V_{wb} \) Wellbore volume, bbl
- \( t_{wb} \) Wellbore storage duration, hr
1.5 Secondary Recovery Processes

**Interfacial tension (IFT)**

\[ \sigma = \frac{rh(p_w - \rho_a)g}{2\cos \theta} \]

**Capillary Pressure in a Tube**

\[ p_c = \frac{2\sigma \cos \theta}{r} \]

**Effective and Relative Permeabilities**

\[ k_w = \frac{q_w B_w \mu_w L}{0.001127 A \Delta p} \]

\[ k_o = \frac{k}{k_r} \]

\[ k_{rw} = \frac{k_w}{k} \]

\[ k_{ro} = \frac{q_o B_o \mu_o L}{0.001127 A \Delta p} \]

**Symbols**

- \( t_D \): Dimensionless time, unitless
- \( p_D \): Dimensionless pressure, unitless
- \( p_r \): Pressure at radius \( r \), or at observation well, psia
- \( N_p \): Cumulative oil produced from well, STB
- \( q_{oil} \): Most recent oil flow rate, STB/D
- \( p \): Pressure or current reservoir pressure, psia
- \( \sigma \): Interfacial tension, dynes/cm
- \( r \): Capillary tube radius, cm
- \( h \): Height of water rise in the capillary, cm
- \( \rho_w \): Water density, g/cm³
- \( \rho_a \): Air density, g/cm³
- \( g \): Gravity acceleration constant, 980 cm/s²
- \( \theta \): Contact angle or wetting angle, degrees
- \( k \): Absolute permeability, md
- \( q_w \): Water flow rate, STB/D
- \( q_o \): Oil flow rate, STB/D
- \( B_w \): Water formation volume factor, bbl/STB
- \( B_o \): Oil formation volume factor, bbl/STB
- \( \mu_w \): Water viscosity, cp
- \( \mu_o \): Oil viscosity, cp
Length of flow path, ft
Cross-sectional area of flow, ft²
Pressure differential, psi

**Empirical Relationship Between Relative Permeabilities and Water Saturation**

\[
\frac{k_w}{k_ro} = a e^{b S_w}
\]

- \(k_ro\) Relative permeability to oil, fraction
- \(k_w\) Relative permeability to water, fraction
- \(a, b\) Empirically derived constants
- \(S_w\) Water saturation, unitless

**Mobility Ratio of Water to Oil**

\[
M_{w,o} = \frac{(\lambda_w)_{Sw}}{(\lambda_o)_{Sw}} = \left(\frac{k_ro}{\mu_o}\right) \left(\frac{\lambda_o}{\lambda_w}\right) \left(\frac{k_w}{\mu_w}\right)
\]

- \(M_{w,o}\) Mobility ratio of water to oil, unitless
- \(\lambda_w\) Mobility of displacing fluid or water behind the front, fraction
- \(\lambda_o\) Mobility of displaced fluid or oil ahead of the front, fraction
- \(S_w\) Residual oil saturation, unitless
- \(S_{wc}\) Connate water saturation, unitless
- \(k_ro\) Relative permeability to oil evaluated at residual oil saturation \(S_w\), fraction
- \(k_w\) Relative permeability to water evaluated at connate water saturation \(S_{wc}\), fraction
- \(\mu_o\) Oil viscosity, cp
- \(\mu_w\) Water viscosity, cp

**Fractional Flow Equation for Water Displacing Oil in a Linear Horizontal System**

\[
f_w = \frac{1}{1 + \left(\frac{k_w}{k_o}\right) \left(\frac{\mu_w}{\mu_o}\right) + \left(\frac{k_ro}{k_w}\right) \left(\frac{\mu_o}{\mu_w}\right)}
\]

- \(f_w\) Fractional flow of water (water cut), bbl/bbl
- \(k_w\) Effective permeability to water, md
- \(k_o\) Effective permeability to oil, md
- \(k_ro\) Relative permeability to oil, fraction
- \(k_w\) Relative permeability to water, fraction
- \(\mu_w\) Water viscosity, cp
- \(\mu_o\) Oil viscosity, cp

**General Form of Fractional Flow Equation**

\[
f_w = \frac{1}{1 + \left(\frac{0.001127 k_w A}{\mu_o q_i} \left(\frac{\partial p}{\partial x} - 0.433 \Delta p \sin \alpha\right)\right) + \left(\frac{k_w}{k_o}\right) \left(\frac{\mu_o}{\mu_w}\right)}
\]

- \(f_w\) Fractional flow of water (water cut), bbl/bbl
- \(k_w\) Effective permeability to water, md
- \(k_o\) Effective permeability to oil, md
- \(\mu_w\) Water viscosity, cp
- \(\mu_o\) Oil viscosity, cp
- \(A\) Cross-sectional area of flow, ft²