Nodal Analysis of Oil and Gas Production Systems
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Jan Dirk Jansen

The MATLAB software accompanying the book can be downloaded from www.tudelft.nl/nodal

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Preface

This book is based on material used in a three-month course for first-year graduate students in the Petroleum Engineering MSc program at Delft University of Technology (TU Delft). This course aims at providing skills in the development and use of mathematical and computer models for flow through the various parts of an oil and gas production system. The underlying idea is that a basic understanding of the numerical implementation of theoretical concepts should be an important element in the education of all engineering students, even though only a few of them will become tool developers and most of them will just use software tools whose inner workings are hidden behind slick user interfaces.

Although the book is primarily intended for classroom use, it may also be of value for practicing engineers or researchers who want to get acquainted with the essentials of nodal analysis or learn more about the underlying theory.* Prerequisites are a basic understanding of physical transport phenomena; calculus, including first-order differential equations; and preferably an acquaintance with hydrocarbon properties and oil and gas well completions.

Questions and exercises are provided at the end of each chapter. Some of those involve computations that can be performed by hand with a calculator, while others require the use of a computer to run the MATLAB routines that accompany the book. MATLAB questions and assignments form an important ingredient of the course, and a basic level in computer programming will therefore be of help to benefit maximally from the book. Hand calculations are primarily seen as a means to obtain quick order-of-magnitude estimates or to verify the results of computer simulations. However, the theory is presented in the book to be understandable without the reader going through the MATLAB exercises. Worked-out answers to all questions are provided in Appendix G.

An early version of this book was based on material written by prof. Peter K. Currie. Since I took over his course in 2001, the text has gradually been rewritten and expanded. During Spring 2011 I revised major parts while teaching a similar course at Stanford University. Very little of the theory described in the text is new, and I acknowledge the contributions of all those who have pioneered the field of nodal analysis and the underlying models for the various production system elements. I have tried to refer to the original sources as far as I am aware of them. Moreover, I acknowledge the contribution of many students and teaching assistants at TU Delft and Stanford who helped to spot errors, inconsistencies, or material that was unclear, both in the text and in the accompanying MATLAB code. No doubt there is still room for further improvement. If you find errors, note missing references, or have other comments, please let me know at j.d.jansen@tudelft.nl.


* Those readers who are just looking for a quick introduction to nodal analysis should read Sections 1.5.1 and 1.6, perform the assignment in Section 1.8 (no MATLAB required), and then jump to Chapter 8, which gives an overview of the most common nodal analysis applications.
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Chapter 1

Introduction

1.1 What Is Covered in This Chapter?
This introductory chapter is meant to clarify the scope and objectives of the book, give a brief overview of the field of production engineering and the relevant hardware, describe the basic concepts used in oil and gas production system modeling, and introduce the method of nodal analysis.

1.2 About This Book

1.2.1 Objectives. Oil and gas wells are essential elements of oil and gas production systems. A production system can be roughly defined as the equipment required to produce hydrocarbons from a subsurface reservoir to the point of sale. The activities related to the conceptual design and the operation of a production system are collectively referred to as production engineering. An important element of production engineering is the quantification of pressures, temperatures, and flow rates inside the production system for various operating conditions, an activity that usually requires some mathematical model of the system, either in the form of written equations or implemented in a computer program. A particular form of modeling, known as nodal analysis, describes the system as a collection of separate elements with distinct flow properties (Brown 1984; Brown and Lea 1985; Beggs 1991) and builds on theory developed in the classic paper of Gilbert (1954). (Note that literature references are listed alphabetically in the References section of this book.) Nodal analysis has proven to be an effective tool for analyzing production system performance, and forms the basis of several commercial and proprietary software packages.

The objectives of this book are:

- To provide an understanding of the concepts used in computer models for the nodal analysis of flow through a production system. This includes understanding the physics, capturing the essential physics in the form of mathematical expressions, and coding these in a computer model.
- To provide basic skills in using a production system computer model to perform production engineering calculations. This concerns, in particular, generating predictions of pressures and flow rates and using them to optimize the design of the production system.
1.2.2 Scope. This book is not explicitly meant to give a complete overview of the field of production engineering, but, rather, to treat some essential concepts in enough detail to achieve the objectives listed above. In particular, it concentrates on the modeling of the flow in a well system, including the near-well reservoir, but does not treat surface facilities in any detail, nor complex networks of multiple wells. Also, the treatment of the physical principles is aimed at explaining the essential concepts but does not completely cover all aspects involved. For example, the book does treat the equations for pressure drops in various elements of a well system and demonstrates how they can be implemented in a simple computer code. However, it does not cover the temperature drops in those elements, because the equations and the computer implementation are analogous to those for pressure drops, and the details can be found in specialized books. Another example concerns the models for multiphase flow in pipes covered in Chapter 4. There, rather than cover the vast range of available models and correlations, the book describes just a few typical models, not necessarily the most accurate ones, to explain the underlying concepts. Understanding the essential concepts in nodal analysis of a well system should provide the readers with enough knowledge to use commercial nodal analysis software for more complex systems, explore the literature in this field, and expand their knowledge as required. Moreover, the scope of this book does not include a detailed description of hardware components. Several other books are available for this purpose, such as Bellarby (2009); various chapters in the SPE Petroleum Engineering Handbook (Lake 2007) also contain good descriptions of hardware components.

1.2.3 Empiricism and First-Principles Modeling. Many textbooks on production engineering present a large number of formulas without their derivation. In the present book, I tried, wherever feasible, to avoid this approach and instead present the derivation of formulas starting from first principles. However, many production engineering calculations are performed with empirical correlations, or at least semiempirical ones with coefficients “tuned” through experiments; in those instances the presentation of equations without derivation is, of course, unavoidable. Moreover, the description of full physical models would sometimes be too involved and outside the scope of this book—e.g., as in the case of fully “mechanistic” multiphase pipe flow models. Therefore, in such cases the book will revert to (semi)empirical approaches.

1.2.4 MATLAB Routines. Many of the equations developed here have been programmed in MATLAB routines that can be downloaded from the website accompanying the book. MATLAB is a programming language for numerical computing with extensive plotting capabilities and a large number of pre-programmed routines for mathematical operations. For an overview and tutorials, see MATLAB (2017). The understanding and use of the routines accompanying the book requires basic programming skills, but no advanced MATLAB knowledge. In particular, the use of compact vector notation as offered by the MATLAB programming language has mostly been avoided, thus somewhat sacrificing computational and notational efficiency for the benefit of readability and educational value. Questions are provided at the end of the chapters. Some of these can be answered through reasoning, others by performing calculations by hand or with a simple calculator, others require the use of MATLAB routines. In line with modern production
engineering practice, hand calculations are seen primarily as a means to obtain quick order-of-magnitude estimates or to verify the results of computer simulations.

1.2.5 Unit Systems and Notation Conventions. Formulas, data, and example calculations will be presented primarily in consistent SI units. Occasionally the corresponding field units are added to allow easy comparison with results from literature or to give the reader a feel for units still used in oil field practice. In the oil industry, the expression “SI units” is often loosely used to indicate both “strict” (consistent) SI units and “allowable” (possibly inconsistent) SI units. The strict units can be subdivided into the seven base SI units (m, kg, s, A, K, mol, and cd) and derived SI units such as N, Pa, °C, or J. Allowable SI units typically include d (day) and a (year), and are defined in the SPE Metric Standard (SPE 1982).

A brief list of conversion factors is given in Appendix A (for a more extensive list, see SPE 1982). In addition, a number of MATLAB “m-files” for unit conversion can be downloaded from the website accompanying the book (web address listed on title page at time of printing). These have a self-explanatory syntax. Thus, e.g., to convert a value of 1,000 psi into Pa, type:

```matlab
» from_psi_to_Pa(1000)
```

which produces the answer

```
ans = 6894757
```

SI units will be used directly in the text. Non-SI units will be enclosed in round brackets when necessary to avoid confusion. To distinguish between temperatures expressed in °C (or °F) and absolute temperatures expressed in K (or °R), absolute temperatures will be labeled with a subscript: \( T_{\text{abs}} \). Following the SPE Style Guide (SPE, 2015), different notations, expressed in either SI or field units, can be used for the same unit. For example, “day” is indicated with “d” in SI units, while it is indicated with “D” in field units. Oil volumes in field units are indicated with “bbl”, but in combined units with “B”, such that oil flow rates in field units are expressed in B/D. Also, products of units in the denominators of fractional expressions are indicated with a “/” in SI units and a “-” in field units. Thus, kg/m\(^3\)-s should be read as kg/(m\(^3\)×s) and lbs/ft\(^3\)-s as lbs/(ft\(^3\)×s). Dimensions will be enclosed in square brackets, as, for example, in \( J \), is expressed in m\(^3\)/s-Pa (B/D-psi-ft) and has dimensions of \([L^3 m^{-1} t]\).”

Dimensions appear as follows, using the SPE Symbols Standard (Lake 2007, Vol. 7, pp. 103-140):

- L length
- m mass
- M money
- n amount of substance
- q electrical charge
- t time
- T temperature

Following the SPE Style Guide (SPE 2015), variables are always written in *italics*, while subscripts are italicized when they represent SPE standard symbols or acronyms; they are written in Roman font when they represent abbreviations. For example, \( p_{\text{mf}} \) and \( p_{\text{sep}} \) are used to indicate ‘manifold pressure’ and ‘separator pressure’ respectively.
1.3 Production Engineering

Fig. 1.1 displays a high-level overview of activities during oil and gas exploration and production, known as E&P. This process diagram, often referred to as the petroleum life cycle model, can of course be refined to display subactivities at deeper levels. The material in this book is of relevance to the production engineering activities during the development and production phases of the petroleum life cycle, in particular to the sub-activities involving field development planning, detailed design of wells and facilities, and operation of wells and facilities.

1.3.1 Development. Unlike what is suggested in Fig. 1.1, the petroleum life cycle is not just a sequential process. In particular, during the design phase, a lot of activities are performed in an iterative fashion. Fig. 1.2, e.g., displays some of the activities involved in designing a well during a field development project, clearly indicating the iterative nature of the process. At a higher level, several cycles of reappraisal (e.g., based on production performance or new seismic data); redevelopment (e.g., through either the recompletion of existing wells or in-fill drilling of new ones); and production may take place during the life of a field. Each of these activities involves aspects of production engineering.

Fig. 1.1—Petroleum life cycle model.

Fig. 1.2—An example of iterative processes during well design in a field development project. Not shown are the links to other iterative activities during the development process such as geological modeling or design of surface facilities.
The key objective during field development is maximizing the economic benefits within the constraints of the project. This optimization process involves comparing multiple development concepts, usually in combination with multiple subsurface models to reflect geological uncertainties. Early cooperation between geophysicists, geologists, reservoir engineers, production engineers, and well engineers, supported by the appropriate organizational structure and systems (software), is essential to achieve the objective.

Traditionally, the concept of production optimization is used in a somewhat narrower context. For example, the textbooks of Brown (1984) and Beggs (1991) focus on optimizing the various components in the flow path from the reservoir to the separator, and elaborate on the detailed analysis of flow in flowlines, chokes, wells, and the near-well section of the reservoir. This book takes a similar approach. All optimization activities require the use of some kind of model of the production system. Traditionally, these models consisted of relatively simple mathematical equations, accessible to hand analysis, sometimes with the aid of charts or tables. Now, the models are usually much more complicated and require the use of a computer.

1.3.2 Production. Apart from its seemingly linear character, Fig. 1.1 has another flaw. It suggests that all phases of the petroleum life cycle are equally important, or that they take similar amounts of time. In reality, the majority of the life of an oil field is spent in the production phase (see Fig. 1.3). Just as during the design phase, various repetitive processes occur during the production phase. This is illustrated in Fig. 1.4, which represents oil and gas production as a feedback control process, involving measurement, modeling, and control. Two major feedback cycles occur, each on its own timescale (see, e.g., Rossi et al. 2000).

- **Daily Production Control.** On a scale of days to weeks, typical input variables are wellhead chokesettings, water-injection pressures, or lift-gas rates. Measured output from the process includes production variables such as tubing-head pressures and oil, gas, and water rates. Control will often be driven by short-term optimization objectives—e.g., production targets or utilization rates of surface facilities. Models of flow through wells and surface facilities can play an important role in the process of optimizing daily production (Bieker et al. 2007). A typical short-term optimization problem is maximizing oil production by distributing a limited amount of lift gas over a number of producing wells.

- **Reservoir Management.** On a time scale of months to years, the production process consists essentially of draining the reservoir. In addition to the variables that control daily production, input includes production engineering activities such as water or gas shutoff, recompletion, stimulation, or even sidetracking or infill drilling.

Fig. 1.3—Petroleum life cycle model emphasizing the importance of the production phase.