Disclaimer
This book was prepared by members of the Society of Petroleum Engineers and their well-qualified colleagues from material published in the recognized technical literature and from their own individual experience and expertise. While the material presented is believed to be based on sound technical knowledge, neither the Society of Petroleum Engineers nor any of the authors or editors herein provide a warranty either expressed or implied in its application. Correspondingly, the discussion of materials, methods, or techniques that may be covered by patents implies no freedom to use such materials, methods, or techniques without permission through appropriate licensing. Nothing described within this book should be construed to lessen the need to apply sound engineering judgment nor to carefully apply accepted engineering practices in the design, implementation, or application of the techniques described herein.
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Foreword

The Society of Petroleum Engineers (SPE) has a vision to “enable the global oil and gas E&P industry to share technical knowledge needed to meet the world’s energy needs in a safe and environmentally responsible manner.” One way of achieving this vision is by sustaining the competency, professionalism, impartiality, and integrity of the personnel within the industry. SPE has responded to this challenge by establishing the SPE Professional Certification Exam (SPEC), which offers members a vehicle to develop their technical competencies and skills across the entire field of petroleum engineering. The SPEC is internationally recognized and represents a high standard of knowledge in different areas of petroleum engineering via an exam that includes engineering fundamentals and complex practical problems.

The SPEC has been offered internationally now for 13 years and is complemented by an SPE short course that gives candidates insight into the range of topics that the exam will cover and the style of questions that they will face. Initially, no specific course manual existed; in most cases, the SPE Petroleum Engineering Handbook series was the main source of reference for the course.

In the summer of 2008, the SPE Engineering Professionalism Committee discussed the idea of writing a book that could be used as a single reference for the SPEC and exam review course. The initial concept of the book was that of a one-stop, go-to reference for future oil and gas industry professionals, with all major concepts, equations, charts, tables, and formulas between its covers. This soon evolved into a “Quick Reference Guide for Petroleum Engineers,” but because the primary intended use for the book was as a reference for the SPEC and US PE Exam, it finally evolved into the guide you are holding today.

The guide has been written for a wide range of audiences and, therefore, will have many applications and uses. It will be of value to university students, recent graduates, and young professionals within the oil and gas industry and academia. However, it is also largely intended for use by experienced professionals who are working on day-to-day projects and require access to a broader scope of petroleum engineering than what falls immediately within their specific areas of expertise.

Along with being of great value before and during examinations, the guide will also be of great use in the workplace. This guide complements the Petroleum Engineering Handbook series by summarizing all of the concepts in a single volume. As a result, there is no need to carry a suitcase full of books on every assignment. The guide is expected to become commonplace in every department and on every desk, platform, or rig in the oil and gas industry. Additionally, the guide is anticipated to be the first-stop reference when oil industry professionals are faced with any upstream or downstream problem.

With this in mind, the SPE Petroleum Engineering Certification and PE License Exam Reference Guide was written in a way that will allow oil industry professionals to apply a formula or equation (that may not be at the forefront of their minds) to their daily processes and procedures without having to cross-reference other texts. The fact that the guide is largely aimed at professionals who have been in the industry for some time allows the user to be familiar with the concepts behind the procedures, so there is no need for a real textbook-style explanation behind their derivation.

In tune with the SPE vision, daily use of the guide by working engineers will increase professional standards and knowledge sharing, thus creating an industry that “meets the world’s energy needs in a safe and environmentally responsible manner.”

Dr. Mohammed Razik Shaikh, SPEC
Acknowledgments


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SPE extends its appreciation to the SPE Engineering Registration Committee for their efforts in revising the guide in 2019. Specific thanks go to Samuel Cappo, Jarrod Sparks, Paul Lammers, Chris Chamblee, Mark Fisk, Weldon Ransbarger, Lucas Moore, Neal Howard, Eric Robertson, Farrukh Hamza, George Stutz, Jared Clark, David Gaudin, Steven Tkach, and other volunteer contributors.
Chapter 1

Reservoir Engineering

1.1 Volume Calculations

Original Oil in Place in Volumetric Undersaturated Oil Reservoirs

Volumetric Method

Above Bubblepoint Pressure

\[ N = \frac{7.758 \times A \times h \times \phi \times (1 - S_w)}{B_o} \]

Below Bubblepoint Pressure

\[ N = \frac{7.758 \times A \times h \times \phi \times (1 - S_w - S_h)}{B_o} \]

- 7,758: Number of barrels per acre-foot, bbl/acre-ft
- \( N \): Original oil in place, STB
- \( A \): Area of the zone, acres
- \( h \): Average net thickness of the zone, ft
- \( \phi \): Porosity, unitless
- \( B_o \): Oil formation volume factor at initial reservoir pressure, bbl/STB
- \( S_w \): Water saturation at initial reservoir conditions, unitless
- \( S_h \): Water saturation at initial reservoir conditions, unitless
Swi
Water saturation at initial reservoir conditions, unitless

g
Gas saturation, unitless

**Material Balance Method Without Water Influx**

**Above Bubblepoint Pressure**

\[
N = \frac{N_p B_o}{B_w \left( c_o S_o + c_g S_g + c_f S_f \right) \Delta p}
\]

- \(N\) Original oil in place, STB
- \(N_p\) Cumulative oil produced, STB
- \(B_o\) Oil formation volume factor, bbl/STB
- \(B_w\) Oil formation volume factor at initial reservoir pressure, bbl/STB
- \(\Delta p\) Change in volumetric reservoir pressure, psi
- \(c_o\) Oil compressibility, psi\(^{-1}\)
- \(c_g\) Water compressibility, psi\(^{-1}\)
- \(c_f\) Formation compressibility, psi\(^{-1}\)
- \(S_o\) Oil saturation, unitless
- \(S_w\) Water saturation at initial reservoir conditions, unitless

**Below Bubblepoint Pressure**

\[
N = \frac{N_p \left[ B_o + (R_p - R_o) B_g \right]}{B_t - B_o}
\]

- \(N\) Original oil in place, STB
- \(N_p\) Cumulative oil produced, STB
- \(B_t\) Two-phase formation volume factor, bbl/STB
- \(B_o\) Two-phase formation volume factor at initial reservoir pressure, bbl/STB
- \(B_g\) Gas formation volume factor, bbl/scf
- \(R_p\) Cumulative produced gas/oil ratio, scf/STB
- \(R_o\) Solution gas/oil ratio at initial reservoir pressure, scf/STB
- \(R_s\) Solution gas/oil ratio, scf/STB
- \(B_o\) Oil formation volume factor, bbl/STB

**Recovery Factor**

\[
RF = \frac{N_p}{N} = \frac{(B_o - B_t)}{B_t + (R_p - R_o) B_g}
\]

- \(RF\) Recovery factor, fraction
- \(N_p\) Cumulative oil produced, STB
- \(N\) Original oil in place, STB
- \(B_t\) Two-phase formation volume factor, bbl/STB
- \(B_o\) Two-phase formation volume factor at initial reservoir pressure, bbl/STB
- \(B_g\) Gas formation volume factor, bbl/scf
- \(R_p\) Cumulative produced gas/oil ratio, scf/STB
- \(R_o\) Solution gas/oil ratio at initial reservoir pressure, scf/STB
- \(R_s\) Solution gas/oil ratio, scf/STB
- \(B_o\) Oil formation volume factor, bbl/STB
Original Oil in Place in Undersaturated Oil Reservoirs With Water Influx

**Volumetric Method**

\[ N = \frac{7.758 \times A \times h \times \phi \times (1 - S_{wi} - S_{or})}{B_{oi}} \]

- 7,758 Number of barrels per acre-foot, bbl/acre-ft
- \( N \) Original oil in place, STB
- \( A \) Area of the zone, acres
- \( h \) Average net thickness of the zone, ft
- \( \phi \) Porosity, unitless
- \( B_{oi} \) Oil formation volume factor at initial reservoir pressure, bbl/STB
- \( S_{wi} \) Water saturation at initial reservoir conditions, unitless
- \( S_{or} \) Residual oil saturation, unitless

**Material Balance Method**

\[ N = \frac{N_p \left[ B_i + (R_p - R_n) B_w \right] - W_p + B_w W_p}{B_i - B_w + B_o \left( c_w S_{wi} + c_f \frac{1 - S_{wi}}{1 - S_{or}} \right) \Delta \bar{p}} \]

- \( N \) Initial oil in place, STB
- \( N_p \) Cumulative oil produced, STB
- \( B_i \) Two-phase formation volume factor, bbl/STB
- \( R_p \) Two-phase formation volume factor at initial reservoir pressure, bbl/STB
- \( B_w \) Water formation volume factor, bbl/STB
- \( B_g \) Gas formation volume factor, bbl/scf
- \( \Delta \bar{p} \) Change in reservoir pressure, psi
- \( c_w \) Water compressibility, psi⁻¹
- \( c_f \) Formation compressibility, psi⁻¹
- \( W_p \) Cumulative water produced, STB
- \( W_w \) Water influx, bbl
- \( R_p \) Cumulative produced gas/oil ratio, scf/STB
- \( R_n \) Solution gas/oil ratio at initial reservoir pressure, scf/STB
- \( S_{wi} \) Water saturation at initial reservoir conditions, unitless
- \( B_{oi} \) Oil formation volume factor at initial reservoir pressure, bbl/STB
- \( R_{so} \) Solution gas/oil ratio, scf/STB

**Oil Unit Recovery Factor (RF)**

(Water drive with no appreciable decline in reservoir pressure)

\[ RF = \frac{N_p}{N} \times \frac{7.758 \times \phi \times (1 - S_{wi} - S_{or})}{B_{oi}} \text{ STB/acre-ft} \]

**Oil Recovery Efficiency (RE)**

(Water drive with no appreciable decline in reservoir pressure)

\[ RE = 100 \times \frac{1 - S_{or} - S_{wi}}{1 - S_{wi}} \% \]

- \( N \) Original oil in place, STB
- \( \phi \) Porosity, unitless
- \( B_{oi} \) Oil formation volume factor at initial reservoir pressure, bbl/STB
Original Oil in Place in Saturated Oil Reservoirs

**Volumetric Method**

\[ N = \frac{7,758 \times A \times h \times \phi \times (1 - S_{\text{sw}o})}{B_{\text{oi}}} \]

- \( N \): Number of barrels per acre-foot, bbl/acre-ft
- \( A \): Area of the oil zone, acres
- \( h \): Average net thickness of the oil zone, ft
- \( \phi \): Average porosity in the oil zone, unitless
- \( B_{\text{oi}} \): Oil formation volume factor at initial reservoir pressure, bbl/STB
- \( S_{\text{sw}o} \): Initial average connate water saturation in the oil zone, unitless

**Material Balance Method (with Water Influx)**

\[ N = \frac{N_p \left[ B_t + (R_p - R_w) B_{\text{oi}} \right] W_e + B_w W_p}{B_t - B_w + \frac{m B_w}{B_{\text{oi}}} (B_t - B_w)} \]

\[ m = \frac{GB_w}{NB_{\text{oi}}} \]

- \( N \): Initial oil in place, STB
- \( N_p \): Cumulative oil produced, STB
- \( B_t \): Oil formation volume factor, bbl/STB
- \( B_{\text{oi}} \): Oil formation volume factor at initial reservoir pressure, bbl/STB
- \( B_t \): Two-phase formation volume factor, bbl/STB
- \( B_w \): Two-phase formation volume factor at initial reservoir pressure, bbl/STB
- \( B_{\text{oi}} \): Water formation volume factor, bbl/STB
- \( B_g \): Gas formation volume factor, bbl/scf
- \( B_w \): Gas formation volume factor at initial reservoir pressure, bbl/scf
- \( W_p \): Cumulative water produced, STB
- \( W_e \): Cumulative water produced, STB
- \( R_p \): Cumulative produced gas/oil ratio, scf/STB
- \( R_w \): Cumulative produced gas/oil ratio at initial reservoir pressure, scf/STB
- \( R_s \): Solution gas/oil ratio at initial reservoir pressure, scf/STB
- \( R_w \): Solution gas/oil ratio, scf/STB
- \( m \): Ratio of initial reservoir free gas volume to initial reservoir oil volume, unitless
- \( G \): Original gas in place, scf

Original Gas in Place in Volumetric Dry Gas, Wet Gas, and Retrograde Gas Condensate Reservoirs

**Volumetric Method**

\[ G = \frac{7,758 \times A \times h \times \phi \times (1 - S_{\text{sw}o})}{B_{\text{oi}}} \]
Alternatively, if $B_g$ is defined as cf/scf where cf is ft$^3$,

$$G = \frac{43,560 \times A \times h \times \phi \times (1 - S_{wi})}{B_g}$$

7,758 Number of barrels per acre-foot, bbl/acre-ft
43,560 Number of cubic feet per acre-foot, cf/acre-ft
$G$ Original gas in place, Mscf (or scf if alternative equation with $B_g$ as cf/scf)
$A$ Area of the zone, acres
$h$ Average net thickness of the zone, ft
$\phi$ Porosity, unitless
$B_g$ Gas formation volume factor at initial reservoir pressure, bbl/Mscf (or cf/scf for alternative equation)
$S_{wi}$ Water saturation at initial reservoir conditions, unitless

**Applying Bulk Reservoir Volume ($V_b$)**

$$G = \frac{43,560 \times V_b \times \phi \times (1 - S_{wi})}{B_g}$$

**Applying Reservoir Pore Volume ($V_p$)**

$$G = \frac{43,560 \times V_p \times (1 - S_{wi})}{B_g}$$

$G$ Original gas in place, scf
$V_b$ Bulk reservoir volume, acre-ft = $A \times h$
$V_p$ Reservoir pore volume, acre-ft = $A \times h \times \phi = V_b \times \phi$
$\phi$ Porosity, unitless
$S_{wi}$ Water saturation at initial reservoir conditions, unitless
$B_g$ Gas formation volume factor at initial reservoir pressure, cf/scf

**Material Balance Method**

$$p_z = p_i \left(1 - \frac{G_p}{G}\right)$$

**Recovery Factor Using “$B_g$” Terms**

$$RF = \frac{G_p}{G} = \frac{(B_g - B_{pi})}{B_g}$$

**Recovery Factor Using “$p/z$” Terms**

$$RF = \frac{G_p}{G} = \frac{p_z - p_i}{p_z}$$

$RF$ Recovery factor, fraction
$G$ Original gas in place, Mscf
$G_p$ Cumulative gas produced, Mscf
$p$ Reservoir pressure (current or abandonment conditions), psia
$p_i$ Initial reservoir pressure, psia
$z$ Gas compressibility factor (current or abandonment conditions), unitless
Gas compressibility factor at initial reservoir pressure, unitless

$z_i$

Gas formation volume factor at initial reservoir pressure, bbl/Mscf

$B_{gi}$

Gas formation volume factor (current or abandonment conditions), bbl/Mscf

$B_g$

Gas Formation Volume Factor

\[ B_g = \frac{p_z z T}{T_p} \]

\[ B_g = \frac{0.00504 z T}{p} \text{ [bbl/scf]} \]

Alternatively, if $B_g$ is defined as cf/scf where cf is ft$^3$:

\[ B_g = \frac{0.02829 z T}{p} \text{ [cf/scf]} \]

$B_g$ Gas formation volume factor, bbl/scf (or cf/scf for alternative equation)

$p_z$ Pressure at standard conditions, or pressure base, psia = 14.7 psia

$T_p$ Temperature at standard conditions, °R ($°R = °F + 460$) = 60°F = 520°R

$z$ Gas compressibility factor or gas deviation factor, unitless

$T$ Reservoir temperature, °R

$p$ Reservoir pressure, psia

Pseudoreduced Pressure and Temperature

\[ p_{pr} = \frac{p}{p_c} \]

\[ T_{pr} = \frac{T}{T_c} \]

$p_{pr}$ Pseudoreduced pressure, unitless

$p$ Pressure, psia

$p_c$ Pseudocritical pressure, psia

$T_{pr}$ Pseudoreduced temperature, unitless

$T$ Temperature, °R ($°R = °F + 460$)

$T_c$ Pseudocritical temperature, °R

Specific Gravity of a Gas

\[ \gamma_g = \frac{\rho_g}{\rho_{air}} = \frac{M_g}{M_{air}} = \frac{M_g}{28.97} \text{ (assumes gas and air obey the ideal-gas law)} \]

also \[ \gamma_g = \frac{M_g}{28.97} \]

\[ M_a = \sum_j \gamma_j M_j \]

$\gamma_g$ Specific gravity of a gas, unitless

$\rho_g$ Density of a gas, lbm/ft$^3$

$\rho_{air}$ Density of air, lbm/ft$^3$

$M_g$ Molecular weight of a gas, lbm/lbm-mol

$M_{air}$ Molecular weight of air (= 28.97 lbm/lbm-mol)
Specific Gravity of a Reservoir Gas for a One-Stage Separation System

\[
\gamma_o = \frac{R\gamma + 4.602\gamma}{R + 133.316\gamma / M_o}
\]

\[
M_o = \frac{5.954}{\gamma_{API} - 8.811}
\]

\[
\gamma_o = \frac{42.43\gamma}{1.008 - \gamma}
\]

\[
\gamma_o = \frac{141.5}{\gamma_{API} + 131.5}
\]

\(\gamma_o\)  Specific gravity of reservoir gas, unitless
\(R_1\)  Primary (high-pressure) separator gas to stock-tank liquid ratio, scf/STB
\(\gamma\)  Specific gravity of primary separator gas, unitless (air = 1.0)
\(\gamma_{API}\)  Specific gravity of stock-tank hydrocarbon liquid in °API
\(\gamma_r\)  Specific gravity of the liquid hydrocarbons, unitless (water = 1.0)
\(M_o\)  Molecular weight of stock-tank liquid (condensate), lbm/lbm-mol

Specific Gravity of a Reservoir Gas for a Three-Stage Separation System

\[
\gamma_o = \frac{R\gamma + 4.602\gamma + R\gamma + R\gamma}{R + (133.316\gamma / M_o) + R + R}
\]

\(\gamma_2\)  Specific gravity of secondary separator gas, unitless
\(\gamma_3\)  Specific gravity of stock-tank gas, unitless
\(R_2\)  Secondary (low-pressure) separator gas to stock-tank liquid ratio, scf/STB
\(R_3\)  Stock-tank gas to stock-tank liquid ratio, scf/STB

Original Gas in Place in Gas Reservoirs With Water Influx

Material Balance Method

\[
G = \frac{G_p B_g - W + B_w W_p}{B_g - B_w}
\]

\(G\)  Initial gas in place, Mscf
\(G_p\)  Cumulative gas produced, Mscf
\(B_g\)  Water formation volume factor, bbl/STB
\(B_w\)  Gas formation volume factor, bbl/Mscf
\(B_w\)  Gas formation volume factor at initial reservoir pressure, bbl/Mscf
\(W_p\)  Cumulative water produced, STB
\(W_e\)  Water influx, bbl

Gas Unit Recovery Factor (RF)

\[
RF = 43,560 \times \phi \times \left( \frac{1 - S_{wi}}{B_g} - S_w \frac{B_w}{B_p} \right) \text{ scf / acre-ft}
\]
Gas Recovery Efficiency (RE)

\[
RE = \frac{100 \left( 1 - \frac{S_{wi}}{B_{wi}} - \frac{S_{gr}}{B_{gr}} \right)}{\left( 1 - \frac{S_{wi}}{B_{wi}} \right)} \times \%
\]

- **RF**: Gas unit recovery factor, scf/acre-ft
- **RE**: Gas recovery efficiency, %
- **φ**: Porosity, unitless
- **S_{wi}**: Water saturation at initial reservoir conditions, unitless
- **S_{gr}**: Residual gas saturation, unitless
- **B_{wi}**: Gas formation volume factor at initial reservoir pressure, bbl/scf
- **B_{gr}**: Gas formation volume factor at abandonment reservoir pressure, bbl/scf

Material Balance Expressed as a Linear Equation (Havlena and Odeh 1963)

\[
F = NE_o + W_eB_o
\]

\[
F = N (E_o + mE_g + E_{fso}) + W_eB_o
\]

\[
F = N \left( B_o + \left( R_p - R_m \right) B_g \right) + W_eB_o
\]

\[
E_o = \left( B_o - B_m \right) + \left( R_{so} - R_m \right) B_o
\]

\[
E_g = B_g \frac{B_o}{B_{gr}} - 1
\]

\[
E_{fso} = (1 + m)B_o \left( \frac{c_wS_{wi} + c_f}{1 - S_{wi}} \right) \Delta p
\]

\[
E_m = E_o + mE_g + E_{fso}
\]

- **F**: Underground withdrawal, bbl
- **E_o**: Oil and solution gas expansion, bbl/STB
- **E_g**: Gas cap expansion, bbl/STB
- **E_{fso}**: Hydrocarbon space reduction, bbl/STB
- **E_m**: Total expansion, bbl/STB
- **N**: Initial oil in place, STB
- **N_p**: Cumulative oil produced, STB
- **W_e**: Cumulative water influx from the aquifer into the reservoir, STB
- **W_o**: Cumulative water produced, STB
- **B_o**: Oil formation volume factor, bbl/STB
- **B_g**: Gas formation volume factor, bbl/scf
- **B_w**: Water formation volume factor, bbl/STB
- **B_{oi}**: Oil formation volume factor at initial reservoir pressure, bbl/STB
- **B_{gi}**: Gas formation volume factor at initial reservoir pressure, bbl/scf
- **Δp**: Change in reservoir pressure, psi
- **c_w**: Water compressibility, psi⁻¹
- **c_f**: Formation compressibility, psi⁻¹
- **S_{wi}**: Water saturation at initial reservoir conditions, unitless
- **R_p**: Cumulative produced gas/oil ratio, scf/STB
- **R_{so}**: Initial solution gas/oil ratio, scf/STB
- **R_s**: Solution gas/oil ratio, scf/STB
- **m**: Ratio of initial reservoir free gas volume to initial reservoir oil volume, unitless