Petroleum Engineering Handbook
Larry W. Lake, Editor-in-Chief

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II Drilling Engineering  Robert F. Mitchell, Editor
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IV Production Operations Engineering  Joe Dunn Clegg, Editor
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Foreword

This 2006 version of SPE’s Petroleum Engineering Handbook is the result of several years of effort by technical editors, copy editors, and authors. It is designed as a handbook rather than a basic text. As such, it will be of most benefit to those with some experience in the industry who require additional information and guidance in areas outside their areas of expertise. Authors for each of the more than 100 chapters were chosen carefully for their experience and expertise. The resulting product of their efforts represents the best current thinking on the various technical subjects covered in the Handbook.

The rate of growth in hydrocarbon extraction technology is continuing at the high level experienced in the last decades of the 20th century. As a result, any static compilation, such as this Handbook, will contain certain information that is out of date at the time of publication. However, many of the concepts and approaches presented will continue to be applicable in your studies, and, by documenting the technology in this way, it provides new professionals an insight into the many factors to be considered in assessing various aspects of a vibrant and dynamic industry.

The Handbook is a continuation of SPE’s primary mission of technology transfer. Its direct descendents are the “Frick” Handbook, published in 1952, and the “Bradley” Handbook, published in 1987. This version is different from the previous in the following ways:

- It has multiple volumes in six different technical areas with more than 100 chapters.
- There is expanded coverage in several areas such as health, safety, and environment.
- It contains entirely new coverage on Drilling Engineering and Emerging and Peripheral Technologies.
- Electronic versions are available in addition to the standard bound volumes.

This Handbook has been a monumental undertaking that is the result of many people’s efforts. I am pleased to single out the contributions of the six volume editors:

General Engineering—John R. Fanchi, Colorado School of Mines
Facilities and Construction Engineering—Kenneth E. Arnold, AMEC Paragon
Production Operations Engineering—Joe D. Clegg, Shell Oil Co., retired
Reservoir Engineering and Petrophysics—Ed Holstein, Exxon Production Co., retired
Emerging and Peripheral Technologies—Hal R. Warner, Arco Oil and Gas, retired

It is to these individuals, along with the authors, the copy editors, and the SPE staff, that accolades for this effort belong. It has been my pleasure to work with and learn from them.

—Larry W. Lake
Preface

You hold in your hand the very first Drilling Engineering volume of the SPE Petroleum Engineering Handbook. This volume is intended to provide a good snapshot of the drilling state of the art at the beginning of the 21st century.

Obviously, the history of well drilling goes back for millennia. The history of “scientific” oilwell drilling had its beginnings at the end of Word War II. Perhaps one indication was that while Petroleum was first established as a Division of the American Inst. of Mining, Metallurgical, and Petroleum Engineers (AIME) in 1922, it was not established as a Branch until 1948. The first SPE reprinted volume of the Petroleum Branch was the 1953 Transactions of the AIME, Petroleum Development and Technology (Vol. 198). This volume had a total of seven papers related to drilling and completion topics, a relatively small proportion of the total of 344 pages.

The first wave of scientific drilling was an era of slide rules and hand calculations. Several references give an idea of the technology level of this era; Developments in Petroleum Engineering by Arthur Lubinski (1987) provides a good overview of the mechanical engineering aspects of drilling, while W.F. Rogers’ Composition and Properties of Oil Well Drilling Fluids (first edition) gives a picture of wellbore hydraulics in 1948. The technology of this era consisted of relatively simple but effective models of very complex phenomena. Former SPE President Claude Hocott once said that any calculation that could not be summarized on a note card would not be useful, and for that era, he was correct. Today, it is difficult to appreciate the tedium of evaluating these simple formulas with a slide rule.

The next wave of scientific drilling introduced a new computational tool—the electronic computer—beginning in the 1970s. Young engineers, who had used primitive computers as part of their university education, were now ready to break Hocott’s one-card rule and delve into the complexity of the phenomena of drilling. As an example of the explosion of knowledge, consider the 1980 Transactions of the Society of Petroleum Engineers (Vol. 269) (note the name change!). The size of the volume nearly doubled to 629 pages, and the number of drilling- and completion-related papers increased 10-fold. To get a feel for the technology level of this era, the textbook Applied Drilling Engineering by A.T. Bourgoyne et al. gives a good overview of the state of the art in 1984.

We are now beginning a third wave of scientific drilling. The days of novel computer application are reaching their twilight years, and a period of evaluation and consolidation is beginning. Computer science and numerical analysis are at a much higher level of accuracy and sophistication today than they were in the 1970s, and many of the technology developments of that era could be re-examined in light of modern techniques. Further, we all recognize that the computer can do far more than just execute numerical calculations. While we can anticipate some of these developments, I suspect we will find that we did not dream big enough.

In an effort like this, the editor quickly recognizes the multidisciplinary nature of the modern drilling process, and his own inadequacies in most of these disciplines. I would like to thank the efforts of this volume’s authors, going above and beyond the call of duty with little financial reward but with, perhaps, a little fame for being the first. I also would like to thank James Bobo for getting this effort off the ground and providing guidance early in the process. I also thank the Editor-in-Chief, Larry W. Lake, for offering me this task and not allowing me to say no.

—Robert F. Mitchell
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Chapter 1
Geomechanics Applied to Drilling Engineering
Dan Moos, GeoMechanics Intl. Inc.

1.1 Introduction
In the early years of oil drilling and production, wells were primarily drilled on land to moderate depths and with relatively minor horizontal offsets, and an empirical understanding of the impact of geological forces and Earth material properties on required drilling practice was developed by region. Successful practices were defined by trial and (sometimes costly and spectacular) error. Once local conditions were understood, it then became possible to drill new wells with a sufficient degree of confidence to guarantee the safety and economic success of further field developments. However, techniques that were successful in one field were not necessarily successful in other fields, and, therefore, the trial-and-error learning process often had to be repeated.

Because wells have become more expensive and complex, both in terms of well geometry (reach and length) and access to deep, high-temperature, high-pore-pressure, and high-stress regimes, it has become clear that the economic success of field developments can only be assured if geology and tectonics are understood and field activities are designed with that understanding. Furthermore, constraints on engineering practice based on environmental and societal requirements necessitate specially designed mud formulations and drilling techniques. Development and application of these solutions depends critically not only on an understanding of the processes that act within the Earth, but also of the impact of these processes on drilling practice. The study of these processes, of the interactions between them and their effect on Earth materials is called geomechanics.

This section of the Handbook is devoted to geomechanics as applied to drilling engineering. As such, it discusses the geological and tectonic effects that can impact the design and successful completion of oil and gas and geothermal wells, and it introduces methods and techniques to characterize those processes and to make recommendations to mitigate their effects. First, we briefly review the concepts of stress and strain, pore pressure, and effective stress. We continue with a brief overview of tectonics and of the origins of forces within the Earth. The purpose is not to cover these subjects exhaustively, but rather to acquaint the reader with this subject sufficiently to understand what follows and to be informed in discussions with geomechanics experts. Then, we discuss the physical properties of Earth materials, including rock strength. With this background, we then focus on issues related to drilling, starting with the impact of far-field stresses on local conditions around the wellbore.
With this as an introduction, we then define the parameters that are required to develop the geomechanical model of a field and review the various techniques with which they can be measured or constrained. Once the geomechanical model has been developed, it can then be used in well design as part of an integrated process to minimize cost and maximize safety.

1.2 Stress, Pore Pressure, and Effective Stress

1.2.1 Definitions and Tectonic Stresses. Forces in the Earth are quantified by means of a stress tensor, in which the individual components are tractions (with dimensions of force per unit area) acting perpendicular or parallel to three planes that are in turn orthogonal to each other. The normals to the three orthogonal planes define a Cartesian coordinate system ($x_1$, $x_2$, and $x_3$). The stress tensor has nine components, each of which has an orientation and a magnitude (see Fig. 1.1a). Three of these components are normal stresses, in which the force is applied perpendicular to the plane (e.g., $S_{11}$ is the stress component acting normal to a plane perpendicular to the $x_1$-axis); the other six are shear stresses, in which the force is applied along the plane in a particular direction (e.g., $S_{12}$ is the force acting in the $x_2$-direction along a plane perpendicular to the $x_1$-axis). In all cases, $S_{ij} = S_{ji}$, which reduces the number of independent stress components to six.

At each point there is a particular stress axes orientation for which all shear stress components are zero, the directions of which are referred to as the “principal stress directions.” The stresses acting along the principal stress axes are called principal stresses. The magnitudes of the principal stresses are $S_1$, $S_2$, and $S_3$, corresponding to the greatest principal stress, the inter-
mediate principal stress, and the least principal stress, respectively. Coordinate transformations between the principal stress tensor and any other arbitrarily oriented stress tensor are accomplished through tensor rotations.

It has been found in most parts of the world, at depths within reach of the drill bit, that the stress acting vertically on a horizontal plane (defined as the vertical stress, $S_v$) is a principal stress. This requires that the other two principal stresses act in a horizontal direction. Because these horizontal stresses almost always have different magnitudes, they are referred to as the greatest horizontal stress, $S_{H_{\text{max}}}$, and the least horizontal stress, $S_{H_{\text{min}}}$ (Fig. 1.1b).

The processes that contribute to the in-situ stress field primarily include plate tectonic driving forces and gravitational loading (see Table 1.1). Plate driving forces cause the motions of the lithospheric plates that form the crust of the Earth. Gravitational loading forces include topographic loads and loads owing to lateral density contrasts and lithospheric buoyancy. These are modified by the locally-acting effects of processes such as volcanism, earthquakes (fault slip), and salt diapirism. Human activities such as mining and fluid extraction or injection can also cause local stress changes. Because the largest components of the stress field (gravitational loading and plate driving stresses) act over large areas, stress orientations and magnitudes in the crust are remarkably uniform (Fig. 1.2). However, local perturbations, both natural and man-made, are important to consider for application of geomechanical analyses to drilling and reservoir engineering (Fig. 1.3).¹

### 1.2.2 Relative Magnitudes of the Principal Stresses in the Earth.

The vertical stress can be the greatest, the intermediate, or the least principal stress. In 1924, Anderson² developed a classification scheme to describe these three possibilities, based on the type of faulting that would occur in each case (Table 1.2 and Fig. 1.4). A normal faulting regime is one in which the vertical stress is the greatest stress. When the vertical stress is the intermediate stress, a strike-
slip regime is indicated. If the vertical stress is the least stress the regime is defined to be reverse. The horizontal stresses at a given depth will be smallest in a normal faulting regime, larger in a strike-slip regime, and greatest in a reverse faulting regime. In general, vertical wells will be progressively less stable as the regime changes from normal to strike-slip to reverse, and consequently will require higher mud weights to drill.

### 1.2.3 Pore Pressure

Pore pressure is the pressure at which the fluid contained within the pore space of a rock is maintained at depth. In the absence of any other processes, the pore pressure is simply equal to the weight of the overlying fluid, in the same way that the total vertical stress is equal to the weight of the overlying fluid and rock (Fig. 1.5). This pressure is often referred to as the “hydrostatic pressure.” A number of processes can cause the pore pressure to be different from hydrostatic pressure. Processes that increase pore pressure include undercompaction caused by rapid burial of low-permeability sediments, lateral compression, release of water from clay minerals caused by heating and compression, expansion of fluids because of heating, fluid density contrasts (centroid and buoyancy effects), and fluid injection (e.g., waterflooding). Processes that decrease pore pressure include fluid shrinkage, unloading, rock dilation, and reservoir depletion.

Because pore pressure and horizontal stresses are interrelated, changes in pore pressure also cause similar changes in stress. While the exact relationship depends on the properties of the reservoir, it is reasonable to assume that the change in horizontal stress is approximately two-thirds of the change in pore pressure (see Eq. 1.1 and Fig. 1.6). This leads to a considerable reduction in leakoff pressure in a depleted reservoir and an increase in horizontal stress where pore pressure increases.

$$\Delta S_{H_{\text{max}}} = \Delta S_{H_{\text{min}}} = \alpha(1 - 2\nu)/(1 - \nu) \Delta P_p,$$
Fig. 1.2—World stress map showing orientations of the greatest horizontal stress, $S_{\text{max}}$, where it has been measured using wellbore breakouts or inferred from earthquakes. Also shown are the boundaries of the major tectonic plates. Colors of the symbols indicate the relative stress magnitudes (light gray, normal; gray, strike-slip; black, reverse or unknown). This figure was produced using software and data available from the World Stress Map Project website.
if \( \nu = 0.25 \) and \( \alpha = 1 \), and

\[
\Delta S_{H_{\max}} = \Delta S_{H_{\min}} = \frac{2}{3} \Delta P_p, \tag{1.1}
\]

where \( \nu \) is Poisson’s ratio, and \( \alpha (= 1 - K_{\text{dry}}/K_{\text{grain}}) \) is the Biot poroelastic coefficient, which varies between zero for a rock that is as stiff as the minerals of which it is composed and one for most sediments, which are much softer than their mineral components. It is important to note that Eq. 1.1 cannot be used to calculate the relationship between pore pressure and stress in the Earth that develops over geological time because in that case the assumptions used to derive the equation are not valid.
1.2.4 Effective Stress. The mathematical relationship between stress and pore pressure is defined in terms of effective stress. Implicitly, the effective stress is that portion of the external load of total stress that is carried by the rock itself. The concept was first applied to the behavior of soils subjected to both externally applied stresses and pore pressure acting within the pore volume in a 1924 paper by Terzaghi as

\[ \sigma_{ij} = S_{ij} - \delta_{ij} P_p, \]

(1.2)

where \( \sigma_{ij} \) is the effective stress, \( P_p \) is the pore pressure, \( \delta_{ij} \) is the Kronecker delta (\( \delta_{ij} = 1, \) if \( i = j, \) \( \delta_{ij} = 0 \) otherwise), and \( S_{ij} \) represents the total stresses, which are defined without reference to pore pressure. While it is sometimes necessary to use a more exact effective stress law in rock (\( \sigma_{ij} = S_{ij} - \delta_{ij} \alpha P_p, \) where \( \alpha \) is Biot’s coefficient and varies between 0 and 1), in most reservoirs it is generally sufficient simply to assume that \( \alpha = 1. \) This reduces the effective stress law to its original form (Eq. 1.2). When expanded, the Terzaghi effective stress law becomes

\[ \sigma_1 = S_1 - P_p, \]
\[ \sigma_2 = S_2 - P_p, \]
and
\[ \sigma_3 = S_3 - P_p, \]

(1.3)
The concept of effective stress is important because it is well known from extensive laboratory experiments (and from theory) that properties such as velocity, porosity, density, resistivity, and strength are all functions of effective stress. Because these properties vary with effective stress, it is therefore possible to determine the effective stress from measurements of physical properties such as velocity or resistivity. This is the basis for most pore-pressure-prediction algorithms. At the same time, effective stress governs the frictional strength of faults and the permeability of fractures.

1.2.5 Constraints on Stress Magnitudes. If rock were infinitely strong and contained no flaws, stresses in the crust could, in theory, achieve any value. However, faults and fractures exist at all scales, and these will slip if the stress difference gets too large. Even intact rock is limited in its ability to sustain stress differences. It is possible to take advantage of these limits when defining a geomechanical model for a field when other data are not available.

Stress Constraints Owing to Frictional Strength. One concept that is very useful in considering stress magnitudes at depth is frictional strength of the crust and the correlative observation that, in many areas of the world, the state of stress in the crust is in equilibrium with its frictional strength. Because the Earth’s crust contains widely distributed faults, fractures, and planar discontinuities at many different scales and orientations, stress magnitudes at depth (specifically, the differences in magnitude between the maximum and minimum principal effective stresses) are limited by the frictional strength of these planar discontinuities. This concept is schematically illustrated in Figs. 1.7a and 1.7b. In the upper part of the figure, a series of randomly oriented fractures and faults is shown. Because this is a two-dimensional (2D) illustration (for simplicity), it is easiest to consider this sketch as a map view of vertical strike-slip faults. In this case, it is the difference between $\sigma_{H_{\text{max}}} (S_{H_{\text{max}}} - P_p)$ and $\sigma_{H_{\text{min}}} (S_{H_{\text{min}}} - P_p)$ that is
limited by the frictional strength of these pre-existing faults. In other words, as $\sigma_{H\text{max}}$ increases with respect to $\sigma_{H\text{min}}$, a subset of these pre-existing faults (shown in light gray) begins to slip as soon as its frictional strength is exceeded. Once that happens, further stress increases are not possible, and this subset of faults becomes critically stressed (i.e., just on the verge of slipping). The lower part of the figure illustrates using a three-dimensional (3D) Mohr diagram, the equivalent 3D case.

The frictional strength of faults can be described in terms of the Coulomb criterion, which states that faults will slip if the ratio of shear to effective normal stress exceeds the coefficient of sliding friction (i.e., $\tau/\sigma_n = \mu$); see Fig. 1.8. Because for essentially all rocks (except some shales) $0.6 < \mu < 1.0$, it is straightforward to compute limiting values of effective stresses using the frictional strength criterion.

This is graphically illustrated using a 3D Mohr diagram as shown in the lower part of Fig. 1.7. 2D Mohr diagrams plot normal stress along the x-axis and shear stress along the y-axis. Any stress state is represented by a half circle that intersects the x-axis at $\sigma = \sigma_3$ and $\sigma = \sigma_1$ and has a radius equal to $(\sigma_1 - \sigma_3)/2$. A 3D Mohr diagram plots three half circles the endpoints of which lie at values equal to the principal stresses and the radii of which are equal to the principal stress differences divided by 2. Planes of any orientation plot within and along the edges of the region between the circles at a position corresponding to the values of the shear and normal stresses resolved on the planes. Planes that contain the $\sigma_2$ plot along the largest circle are first to reach a critical equilibrium.
The critically stressed (light gray) faults in the upper part of the figure correspond to the points (also shown in light gray) in the Mohr diagram, which have ratios of shear to effective normal stress between 0.6 and 1.0. It is clear in the Mohr diagram that for a given value of $\sigma_{H\text{min}}$, there is a maximum value of $\sigma_{H\text{max}}$ established by the frictional strength of pre-existing faults (the Mohr circle cannot extend past the line defined by the maximum frictional strength).

The values of $S_1$ and $S_3$ corresponding to the situation illustrated in Fig. 1.7 are defined by

$$\frac{\sigma_1}{\sigma_3} = \frac{(S_1 - P_p)}{(S_3 - P_p)} = \left[\left(\mu^2 + 1\right)^{1/2} + \mu\right]^2 = f(\mu).................................. (1.4)$$

That is, it is the effective normal stress on the fault (the total stress minus the pore pressure) that limits the magnitude of the shear stress. Numerous in-situ stress measurements have demonstrated that the crust is in frictional equilibrium in many locations around the world (Fig. 1.9). This being the case, if one wished to predict stress differences in-situ with Eq. 1.4, one would use Anderson’s faulting theory to determine which principal stress (i.e., $S_{H\text{max}}$, $S_{H\text{min}}$, or $S_v$) corresponds to $S_1$ or $S_3$, depending of course on whether it is a normal, strike-slip, or reverse-faulting environment, and then utilize appropriate values for $S_v$ and $P_p$ (the situation is more complex in strike-slip areas because $S_v$ corresponds to neither $S_1$ nor $S_3$). Regardless of whether the state of stress in a given sedimentary basin reflects the frictional strength of pre-existing faults, the importance of the concept illustrated in Fig. 1.7 is that at any given depth and pore pressure, once we have determined the magnitude of the least principal effective stress using minifracs or leakoff tests ($\sigma_{H\text{min}}$ in a normal or strike-slip faulting case), there is only a finite range of values that are physically possible for $\sigma_{H\text{max}}$.

Eq. 1.4 defines the upper limit of the ratio of effective maximum to effective minimum in-situ stress that is possible before triggering slip on a pre-existing, well-oriented fault. The in-situ effective stress ratio can never be larger than this limiting ratio. Therefore, all possible stress states must obey the relationship that the effective stress ratios must lie between 1 and the limit defined by fault slip as shown in Eq. 1.5.

$$1 \leq \frac{\sigma_1}{\sigma_2} \leq \frac{\sigma_1}{\sigma_3} \leq \left[\left(\mu^2 + 1\right)^{1/2} + \mu\right]^2 = f(\mu). ...................................... (1.5)$$

These equations can be used along with the Andersonian definitions of the different faulting regimes (Table 1.1) to derive a stress polygon, as shown in Fig. 1.10. These figures are constructed as plots at a single depth of $S_{H\text{max}}$ vs. $S_{H\text{min}}$. The shaded region is the range of
allowable values of these stresses. By the definitions of \( S_{H\text{max}} \) and \( S_{H\text{min}} \), the allowable stresses lie above the line for which \( S_{H\text{max}} = S_{H\text{min}} \). Along with the pore pressure, \( S_v \), shown as the black dot on the \( S_{H\text{max}} = S_{H\text{min}} \) line, defines the upper limit of \( S_{H\text{max}} \) [the horizontal line at the top of the polygon, for which \( \sigma_{H\text{max}}/\sigma_v = f(\mu) \)], and the lower limit of \( S_{H\text{min}} \) [the vertical line on the lower left of the polygon, for which \( \sigma_v/\sigma_{H\text{min}} = f(\mu) \)]. The third region is constrained by the difference in the horizontal stress magnitudes [i.e., \( \sigma_{H\text{max}}/\sigma_{H\text{min}} < f(\mu) \)]. The larger the magnitude of \( S_v \), the larger the range of possible stress values; however, as the pore pressure increases, the polygon shrinks, until at the limit when \( P_p = S_v \), all three stresses are equal.

It is important to emphasize that the stress limit defined by frictional faulting theory is just that—a limit—and provides a constraint only. The stress state can be anywhere within and along the boundary of the stress polygon. As discussed at length later, the techniques used for quantifying in-situ stress magnitudes are not model based, but instead depend on measurements, calculations, and direct observations of wellbore failure in already-drilled wells in the region of interest. These techniques have proved to be sufficiently robust that they can be used to make accurate predictions of wellbore failure (and determination of the steps needed to prevent failure) with a reasonable degree of confidence.

**Stress Constraints Owing to Shear-Enhanced Compaction.** In weak, young sediments, compaction begins to occur before the stress difference is large enough to reach frictional equilibrium. Therefore, rather than being at the limit constrained by the frictional strength of faults, the stresses will be in equilibrium with the compaction state of the material. Specifically, the porosity and stress state will be in equilibrium and lie along a compactional end cap. The physics of this process is discussed in the section on rock properties of this chapter.
Constraints, based on compaction, define another stress polygon similar to the one shown in Fig. 1.10. It is likely that in regions such as the Gulf of Mexico, and in younger sediments worldwide where compaction is the predominant mode of deformation, this is the current in-situ condition. Unfortunately, while end-cap compaction has been studied in the laboratory for biaxial stress states \((\sigma_1 > \sigma_2 = \sigma_3)\), there has been little laboratory work using polyaxial stresses \((\sigma_1 \neq \sigma_2 \neq \sigma_3)\), and there have been relatively few published attempts to make stress predictions using end-cap models. Also, it is important to apply end-cap analyses only where materials lie along a compaction curve, and not to apply these models to overcompacted or diagenetically modified rocks. If the material lies anywhere inside the region bounded by its porosity-controlled end cap, this constraint can be used only to provide a limit on stress differences.

1.3 Rock Properties

1.3.1 Deformation of Rocks—Elasticity. To first order, most rocks obey the laws of linear elasticity. That is, for small strains, the elements of the stress and strain tensors are related through

\[
\sigma_{ij} = M_{ijkl} \varepsilon_{kl},
\]

where

\[
\varepsilon_{kl} = \Delta l_k / l_i,
\]

\text{................................. (1.6)}
In other words, the stress required to cause a given strain, or normalized length change ($\Delta l/l$), is linearly related to the magnitude of the deformation and proportional to the stiffnesses (or moduli), $M_{ijkl}$. Furthermore, the strain response occurs instantaneously as soon as the stress is applied, and it is reversible—that is, after removal of a load, the material will be in the same state as it was before the load was applied. A plot of stress vs. strain for a laboratory experiment conducted on rock that obeys such a law is a straight line with slope equal to the modulus. However, in real rocks, the moduli increase as a function of effective stress, particularly at low stress. Some of this increase is reversible (nonlinear elasticity), but some of it is irreversible (plasticity or end-cap compaction). To make matters even more complicated, rocks also exhibit time-dependent behavior, so that an instantaneous stress change elicits both an instantaneous and a time-dependent response. These anelastic effects can be seen in laboratory experiments, as shown in Fig. 1.11.

At the top of Fig. 1.11 is shown the stress as a function of time applied in the laboratory to two samples of an upper Miocene turbidite. As in most experiments of this type, a cylindrical rock sample is jacketed with an impermeable soft sleeve and placed in a fluid-filled pressure cell. The fluid pressure surrounding the sample is increased slowly, and the fluid pressure (confining stress) and sample axial and circumferential strains are measured. To identify the various deformation processes that occur in this unconsolidated sand, the stress is slowly increased at a constant rate and then held constant until the sample stops deforming. Then the pressure is decreased to approximately half of the previous maximum pressure. After that, the pressure is increased at the same rate until the next pressure step is completed. This process continues until the desired maximum pressure is achieved, and then the sample is slowly unloaded and removed from the pressure cell.

All aspects of typical rock behavior can be seen in the stress-strain curve plotted on the bottom of Fig. 1.11. At low pressure, the sample is soft, and there is a rapid increase of stiffness with pressure (nonlinear elasticity) owing to crack closure, as well as an increase in stiffness caused by irreversible compaction. Once the pressure increase stops, the sample continues to deform, with deformation rate decreasing with time (time-dependent creep). The sample is stiffer during unloading than during loading, and during this phase of the experiment, it essentially behaves as a linear elastic material; the permanent strains during loading and creep that occurred through plastic/viscous deformation mechanisms are not recoverable. Reloading follows the (purely elastic) unloading path until the maximum previous pressure has been reached, after which additional plastic deformation begins to occur again as the material resumes following the compaction curve. All of these effects can be seen in situ, including the difference between the loading and unloading response.

Measurements of $P$-wave and $S$-wave velocity made on this sample during the experiment by measuring the time of flight of pulses transmitted axially along the sample were used to calculate the dynamic shear ($G$) and bulk ($K$) moduli with Eq. 1.7. The implications of the results for pore pressure prediction are discussed later in this chapter.

$$G = \rho V_s^2; \quad K = \rho V_p^2 - 4G/3.$$

The dynamic bulk modulus calculated from the velocity measurements is higher than the moduli computed from the slopes of the unloading/reloading curves, which in turn are larger than the modulus calculated from the slope of the loading curve. This dispersion (frequency-dependence of the moduli) also is typical of reservoir rocks, and it is the justification for empirical corrections applied to sonic log data to convert from the dynamic moduli measured by the sonic log to static moduli that are used to model reservoir response. However, it is important to realize that there are two different “static” moduli—a “compaction modulus,” the slope of the loading curve, which includes plastic effects, and a “static elastic modulus” mea-
sured by unloading/reloading, which is truly elastic. It is critical when measuring material response in the lab to differentiate between these two and to use the appropriate one for in-situ modeling—the elastic unloading modulus when no compaction is occurring, for example when pore pressure is increased by injection during waterflooding, and the compaction modulus when modeling, for example, very large depletions in weak reservoirs.

These considerations can become very important when modeling and predicting how the wellbore will respond during and after drilling. In the discussion of wellbore stability that follows, however, we will assume that the rock is purely elastic and only briefly discuss the implications of more complicated rheological models.
1.3.2 Compaction and End-Cap Plasticity. When rocks are loaded past a certain point, they will no longer behave elastically. If the load is approximately isotropic ($\sigma_1 \approx \sigma_2 \approx \sigma_3$), the rock will begin to compact and lose volume, primarily because of a decrease in porosity. This process is referred to as shear-enhanced compaction because, in general, the effect occurs at lower mean stress as the shear stress increases. Fig. 1.12 shows a plot of the shear stress as a function of mean stress for a variety of rocks, labeled for use in a normal faulting regime where $S_v$ is $S_1$. Compaction trends are shown as arcs bounding the data from the right, and they define end caps of the stress regime within which the rock at a given porosity can exist. Values of porosity decrease as the end caps move outward, owing to material compaction that is caused by the increase in confining stress. The shapes of the end caps for any porosity depend on the form of the relationship between the mean stress at the compaction limit and the shear stress. In many studies, the shape of the end-cap is assumed to be elliptical. At any point along an end cap, the porosity is in equilibrium with the state of effective stress.

In unconsolidated materials, shear-enhanced compaction begins at zero confining stress as soon as the material begins to be loaded (see Fig. 1.11). In situ, this compaction is the primary cause of the increase in stiffness and decrease in porosity of sediments with burial. The assumption inherent in all standard pore-pressure-prediction algorithms that rock properties are uniquely related to the effective stress is equivalent to assuming that the rock in situ lies along a compaction trend defined by an end cap.

If the mean effective stress decreases (for example, because of erosion) or the pore pressure increases, the rock becomes overcompacted. When this occurs, its porosity is no longer in equilibrium with the end cap, and it will behave elastically, as occurred during the unloading stages of the experiment shown in Fig. 1.11.
When a differential load is applied (e.g., $\sigma_1 > \sigma_2 = \sigma_3$), eventually the maximum stress ($\sigma_1$) will get so large that the sample either will begin to yield through a process of distributed deformation or will fail because of shear localization and the creation of a brittle failure surface (a fault). In Fig. 1.12, the data at the left edge of the plot lie along a limit in the ratio of shear stress to mean stress that is defined by the onset of brittle failure or plastic yielding by shear localization, as discussed next.

1.3.3 Failure Models and Rock Strength. Rock strength models that define stress states at which brittle failure occurs follow stress trajectories that lie along the left edge of the data shown in Fig. 1.12. It is clear that the ability of a rock to carry differential stress increases with confining stress. To establish the exact relationship, rock strength tests are conducted at a number of confining pressures. In these tests, a jacketed, cylindrical sample is loaded into a pressure vessel, a constant confining pressure is established, and an axial load is applied by means of a hydraulic ram. The load is increased slowly by driving the ram at a constant rate, monitoring axial and circumferential strains and maintaining a constant confining pressure, until the sample fails or yields. An example of an axial stress vs. axial strain plot from a typical triaxial stress experiment is shown in Fig. 1.13.

One criterion to define the stress state at failure is the 2D linear Mohr-Coulomb criterion. The Mohr-Coulomb criterion defines a linear relationship between the stress difference at failure and the confining stress using two parameters: $S_o$, the cohesion (or $C_o$, the unconfined
compressive strength) and $\Phi$, the angle or $\mu_i$, the coefficient of internal friction, where $\mu_i = \tan \Phi$. The equation that defines the 2D linear criterion is $\tau = S_o + \mu_i \sigma_n$. These parameters can be derived from triaxial strength tests on cylindrical cores by measuring the stress at failure as a function of confining pressure.

Fig. 1.14 shows graphically how the Mohr-Coulomb parameters are derived. The upper plot shows a series of Mohr circles, with $x$-intercepts $\sigma_1$ and $\sigma_3$ at failure and diameter $\sigma_1 - \sigma_3$, in a plot of shear stress to effective normal stress. The failure line with slope $\mu_i$ and intercept $S_o$ that just touches each of the circles defines the parameters of the 2D linear Mohr-Coulomb strength criterion for this material. $C_o$ is the value of $\sigma_1$ for $\sigma_3 = 0$ of the circle that just touches the failure line. The lower plot graphs $\sigma_1$ vs. $\sigma_3$ and can be used to derive $C_o$ directly.

Some of the other strength criteria include the Hoek and Brown (HB) criterion, which, like the Mohr-Coulomb criterion, is 2D and depends only on knowledge of $\sigma_1$ and $\sigma_3$, but which uses three parameters to describe a curved failure surface and, thus, can better fit Mohr envelopes than can linear criteria. The Tresca criterion is a simplified form of the linearized Mohr-Coulomb criterion in which $\mu_i = 0$. It is rarely used in rocks and is more commonly applied to
metals, which have a yield point but do not strengthen with confining pressure. Other failure
criteria, such as Drucker Prager (inscribed and circumscribed, both extensions of the von Mises
criterion) and Weibols and Cook incorporate the dependence of rock strength on the intermediate
principal stress, $\sigma_2$, but require true polyaxial rock strength measurements that have $\sigma_1 > \sigma_2$
$> \sigma_3$ and are difficult to carry out. The Modified Lade Criterion has considerable advantages,
in that it, too, is a 3D strength criterion but requires only two empirical constants, equivalent
to $C_o$ and $\mu_s$. Thus, it can be calibrated in the same way as the simpler 2D Mohr-Coulomb
failure criterion, but because it is fully 3D, it is the preferred criterion for analysis of wellbore
stability.

1.3.4 Single-Sample Testing. Because triaxial tests are so difficult and time-consuming to carry
out, and because of the amount of core required and the difficulty in finding samples that are similar enough to be considered identical, it is common to attempt to reduce the number of
tests requiring core preparation. One method is simply to carry out a uniaxial strength test in which the confining pressure is zero. This requires a much simpler apparatus; in fact, the sample
does not even have to be jacketed, although this is recommended. By definition, the axial stress at failure in a uniaxial test is a direct measure of $C_o$. Unfortunately, unconfined samples
can fail in a variety of ways that do not provide a good measure of $C_o$ for use with a Mohr-
Coulomb model. Furthermore, it is impossible to measure $\mu_s$ using one test unless a clearly
defined failure surface is produced, the angle of which with respect to the loading axis can be measured. For these reasons, a series of triaxial tests is preferred.

An alternative method that does require testing in a triaxial cell is to carry out a series of
tests on a single sample. The process proceeds by establishing a low-confining pressure and then increasing the axial stress until the sample just begins to yield. At that point, the test is stopped, the confining pressure is increased, and again the axial stress is increased until yielding occurs. In comparisons of this method against multiple triaxial tests, it is often the case that the yield stress derived from the multistage test is systematically lower, and the internal friction is also systematically lower, than the stress at failure and the internal friction derived from the triaxial tests. This is because, once the initial yielding has begun, the sample is already damaged and thus is weaker than it would be had this not occurred. However, by using this method, it may be possible to characterize the yield envelope of a plastic rock.

1.3.5 Scratch and Penetrometer Testing. A number of techniques have been developed to replace or augment triaxial tests to measure the strength properties of rocks. One such technique, which has a demonstrated ability to provide continuous, fine-scale measurements of both elastic and strength properties, is the scratch test. This test involves driving a sharp cutter across a rock surface. By monitoring the vertical and lateral forces required to maintain a certain depth of cut, it is possible to determine the uniaxial compressive strength, $C_o$. The Young’s modulus, $E$, can also be estimated in some cases. Fig. 1.15 shows a comparison of $C_o$
derived by scratch testing to laboratory core measurements and log-derived $C_o$. The results are quite similar.

The advantage of scratch testing is that no special core preparation is required. This is in contrast to the extensive preparations required prior to triaxial testing. The test can be conducted either in the lab or, in principle, on the rig, almost immediately after recovery of core material. No significant damage occurs to the core, which makes this a very attractive substitute for triaxial testing when little material is available. In fact, research is now under way to evaluate the feasibility of designing a downhole tool to carry out this analysis.

In a penetrometer test, a blunt probe is pressed against the surface of a rock sample using continuously increasing pressure. The unconfined compressive strength is then computed from the pressure required to fracture the sample. As in the case of scratch testing, no special sam-
ple preparation is required. In fact, any sample shape can be used for a penetrometer test, and even irregular rock fragments such as those recovered from intervals of wellbore enlargement because of compressive shear failure can be tested. Recently, methods have been developed to apply penetrometer tests to drill cuttings. Although these have not been widely used, they show considerable promise, and in the future they may become an important component of the measurement suite required to carry out wellbore stability analysis in real time.

1.3.6 Estimating Strength Parameters From Other Data. It is relatively straightforward to estimate $C_o$ using measurements that can be obtained at the rig site. Log or logging while drilling (LWD) measurements of porosity, elastic modulus, velocity, and even gamma ray activity (GR) have all been used to estimate strength. For example, Fig. 1.16 shows a plot of $C_o$ computed from $P$-wave modulus ($\rho_b V_p^2$) for Hemlock sands (Cook Inlet, Alaska).

It is possible to develop an empirical relationship between any log parameter (even GR—see Fig. 1.17) and $C_o$ or internal friction $\mu_i$. Measurements of cation exchange capacity (CEC) and $P$-wave velocity have both been used for this purpose.

Because velocity, porosity, and GR can be acquired either using LWD or by measurements on cuttings carried out at the drilling rig from which CEC can also be derived, it is now possible to produce a strength log almost in real time. It is important, however, to recognize that different rock types will have very different log-strength relationships, based on their lithology (sand/shale, limestone, dolomite), age, history, and consolidation state. Therefore, it is important to be careful to avoid applying to one rock type a relationship calibrated for another.

1.4 Elastic Wellbore Stress Concentration

1.4.1 Stresses Around a Vertical Well. For a vertical well drilled in a homogeneous and isotropic elastic rock in which one principal stress (the overburden stress, $S_v$) is parallel to the wellbore axis, the effective hoop stress, $\sigma_{th}$, at the wall of a cylindrical wellbore is given by Eq. 1.8.
Here, $\theta$ is measured from the azimuth of the maximum horizontal stress, $S_{\text{Hmax}}$, $S_{\text{Hmin}}$ is the minimum horizontal stress; $P_p$ is the pore pressure; $\Delta P$ is the difference between the wellbore pressure (mud weight) and the pore pressure, and $\sigma^{\Delta T}$ is the thermal stress induced by cooling of the wellbore by $\Delta T$. If there is no strain in the axial direction, the effective stress acting parallel to the wellbore axis ($\sigma_{zz}$) is

$$\sigma_{zz} = S_v - 2\nu(S_{\text{Hmax}} - S_{\text{Hmin}}) \cos 2\theta - P_p - \sigma^{\Delta T}, \quad \text{........................................... (1.9)}$$
where $\nu$ is Poisson’s ratio. At the point of minimum compression around the wellbore (i.e., at $\theta = 0$, parallel to $S_{H\max}$), Eq. 1.8 reduces to

$$\sigma_{\theta\theta}^{\text{min}} = 3S_{H\min} - S_{H\max} - 2P_p - \Delta P - \sigma^{NT}.$$ ..................................... (1.10)

At the point of maximum stress concentration around the wellbore (i.e., at $\theta = 90^\circ$, parallel to $S_{H\min}$),

$$\sigma_{\theta\theta}^{\text{max}} = 3S_{H\max} - S_{H\min} - 2P_p - \Delta P - \sigma^{NT}.$$ ..................................... (1.11)

The equations for $\sigma_{\theta\theta}$ and $\sigma_{zz}$ are illustrated in Fig. 1.18 for a strike-slip/normal faulting stress regime ($S_{H\max} \sim S_v > S_{H\min}$) at a depth of 5 km, where the pore pressure is hydrostatic and both $\Delta P$ and $\sigma^{NT}$ are assumed to be zero for simplicity. As indicated in Eq. 1.11 and illustrated in Fig. 1.18, at the point of maximum compression around the wellbore, the maximum principal
horizontal stress is amplified appreciably. If the stress concentration is high enough, it can exceed the rock’s strength, and the rock will fail in compression. Compressive failures that form in the region around the wellbore where the stress concentration is greatest are commonly called stress-induced wellbore breakouts.

For the stress state assumed in Fig. 1.18, the stress concentration is close to zero at the azimuth of the maximum horizontal stress, $S_{H\text{max}}$. This is because a strike-slip faulting stress state was used for these calculations. It can be straightforwardly shown that in a strike-slip stress state in which the horizontal stress difference is in equilibrium with the strength of vertical strike-slip faults...
Substituting this relation into Eq. 1.10 demonstrates that $\sigma_{\theta \theta}^{\text{min}} \approx 0$, and it is easy for the wellbore to fail in tension, especially if $\Delta P$ and $\Delta T$ are greater than zero. Because the horizontal stress difference is smaller in a normal or reverse-faulting stress state than for a strike-slip stress state, tensile failure is less likely in these faulting regimes unless a wellbore is inclined.

To consider the potential for wellbore failure when a wellbore is inclined to the principal stresses, it is necessary to take into account the magnitudes and the orientations of the principal far-field stresses. Once these stress components are determined, in order to know whether a wellbore is likely to fail, the magnitudes of the stresses around the wellbore must be computed and the results considered in the context of a formal failure criterion. Because the equations that describe the stress concentration around a well inclined to the principal stress axes are complicated, they are usually solved using a computer application designed for the purpose.

The wellbore stress concentration decreases as a function of radial distance from the wellbore wall. Thus, the zone of failed rock will only extend to a certain depth away from the well. Once the rock has failed, however, the stresses are re-concentrated around the now broken-out wellbore, and it is possible (depending on the residual strength of the failed rock, which determines whether it can support stress) that additional failure will occur. One important thing to keep in mind is that even if the rock has failed, it may not lead to drilling difficulties.

1.4.2 Compressive Wellbore Failure. Stress-induced wellbore breakouts form because of compressive wellbore failure when the compressive strength of the rock is exceeded in the region of maximum compressive stress around a wellbore (Fig. 1.18). If the rock inside the breakout has no residual strength, the failed rock falls into the wellbore and gets washed out of the hole. The shape of these cuttings can be diagnostic of the mode of wellbore failure. Assuming (for the sake of discussion) that a Mohr-Coulomb failure criterion is appropriate for relatively brittle rocks, Fig. 1.19 shows the potential shear failure surfaces for the indicated stress field (left), and the zone of initial failure for a given cohesive strength, $S_o$ (right). Comparison of the wellbore cross sections with the failure trajectories suggests that the surface of some breakouts is defined by a single shear fracture. It also has been demonstrated that wider and deeper breakouts will form as the maximum horizontal stress increases or as rock strength or mud weight decreases. While there is an increase in the stress concentration at the back of the breakout once it forms, any additional failure caused by that new stress concentration will result in an increase in breakout depth but will not change the width.

In a vertical well, breakouts are centered at the azimuth of minimum horizontal stress $S_{H\text{min}}$ because this is where the compressive hoop stress is greatest. Hence, one can directly deduce the orientation of the in-situ stress tensor from the observation of breakouts. In inclined wells or in vertical wells where one principal stress axis is not parallel to the wellbore, breakout orientations are a function of both the orientations and the magnitudes of the in-situ stresses. Breakouts also may rotate in wells that intersect active shear planes. In both cases, while it is not possible to determine the stress orientation without additional information, it is often possible to determine one or more stress magnitudes.

1.4.3 Tensile Wellbore Failure. It is well known that if a vertical wellbore is pressurized, a hydraulic fracture will form at the azimuth of the maximum horizontal stress $S_{H\text{max}}$. In some cases, the natural stress state, perhaps aided by drilling-related perturbations such as high mud weight, causes the wellbore wall to fail in tension, generating drilling-induced tensile wall fractures (DITWFs), as previously discussed for a vertical well in a strike-slip faulting environment. These fractures occur only at the wellbore wall (owing to the local stress concentration) and do not propagate any significant distance into the formation. They form 90° from the az-
imuths of wellbore breakouts, and in vertical wells they indicate the azimuth of the maximum horizontal stress. As in the case of breakouts, tensile fractures in wells inclined to the principal stresses form at orientations that are a function of the stress magnitude as well as its orientation. In such cases, tensile fractures are inclined with respect to the wellbore axis, thus providing a clear indication that the stresses are not parallel and perpendicular to the well.

1.4.4 Detecting Wellbore Breakouts and Tensile Fractures. Wellbore breakouts were first identified by Gough and Bell using 4-arm, magnetically oriented caliper logs acquired with Schlumberger dipmeters. However, to use this information for stress analysis, breakouts must be distinguished from other enlargements such as washouts (in which the entire hole is enlarged) and keyseats (caused by pipe wear or other drilling-related wellbore damage). The criteria, illustrated in Fig. 1.20, used to distinguish stress-induced wellbore breakouts from drilling-induced features are as follows. First, when the caliper tool encounters a breakout, the tool should stop rotating in the well, because it should be engaged in the enlargement. Second, the small diameter measured by the caliper must be equal to the bit size. Third, in the case of an inclined well, the direction in which the wellbore is enlarged should not be the same as the direction of wellbore deviation. Finally, neither caliper diameter should be smaller than the bit size, as can occur in zones of keyseats owing to an associated off-centered tool. Failure to utilize criteria such as these can result in interpreting washouts and keyseats as wellbore breakouts.

While breakouts can be detected and used to determine stress orientation in many wells if 4-arm caliper data are carefully analyzed using rigorous criteria, truly unambiguous identification of breakouts requires the interactive analysis of data from full-wellbore scanning tools such as acoustic televiewers, which generate wellbore images that allow a much more detailed investigation of the wellbore wall (Fig. 1.21). These image data have the advantage over caliper data in that it is possible in images to

1. Study detailed variations of breakout orientation with depth.
2. Analyze the precise span of the wellbore’s circumference which has failed using wellbore cross sections based on the time of flight of the acoustic pulse.
3. Unambiguously distinguish stress-induced breakouts from keyseats and washouts. Although electrical image logs can also be used for wellbore failure analysis, it is more difficult to detect and characterize wellbore breakouts in electrical images than in acoustic images.

With the advent of 6-arm, oriented calipers, both those associated with electrical imaging tools and those that are run independently, it is now possible to utilize such data to define the shape of a well and identify oriented enlargements such as those caused by breakouts. To do so, however, these logs must be run in combination with orientation devices. As with 4-arm
caliper (dipmeter) data, strict criteria must be defined before using these data to determine stress orientation. An example of a case in which televiewer data was available to validate a 6-arm caliper analysis is shown in Fig. 1.22. Here, the wellbore cross section provided by the acoustic time-of-flight data from an acoustic wellbore imaging tool is shown on the right, along with radial lines indicating the azimuthal extent of a wellbore breakout (courtesy GeoMechanics Intl. Inc.).

![Image of wellbore cross section and 6-arm caliper data](image)

**Fig. 1.22**—Comparisons of 6-arm caliper data with wellbore cross sections from acoustic televiewer data from within the same interval reveal clearly that the 6-arm tool can be used to detect and orient breakouts.

caliper (dipmeter) data, strict criteria must be defined before using these data to determine stress orientation. An example of a case in which televiewer data was available to validate a 6-arm caliper analysis is shown in Fig. 1.22. Here, the wellbore cross section provided by the acoustic time-of-flight information shows enlargements in the precise orientations of the enlarged parts of the hole detected using 6-arm calipers. It is not possible to constrain breakout widths using 6-arm calipers because the orientation scatter in that data reflects only the variation in position of the centers of the caliper pads where breakouts were detected. Thus, the widths of these two cross sections have little relationship to each other.

Tensile fractures can most easily be seen in electrical image logs (see Fig. 1.23), whereas in acoustic images, they are most often seen when they are associated with fluid losses (e.g.,
In some very rare cases, wellbores will enlarge in the direction in which tensile fractures are created by excessive amounts of wellbore cooling or extremely high mud weights. This effect has been documented using image logs in cases where stress orientations obtained from caliper logs were interpreted to indicate 90° shifts in stress orientation across bed boundaries. Even if televiewer data are available, enlargements in the direction in which tensile fractures develop can be mistaken for breakouts unless the data are studied carefully.

The cracks seen in Fig. 1.23 occur on both sides of the wellbore at the orientation of the maximum horizontal principal stress in the region, and similar cracks are seen over a ~200-m-long interval of the relatively vertical section of this well. These cracks are principally the result of the natural stress state combined with the additional effects of excess wellbore pressure and cooling, and thus the state of stress implied by the occurrence of these fractures is strike-slip ($S_{H\text{max}} > S_{v} > S_{H\text{min}}$).

1.4.5 Effects of Mud Weight and Temperature on the Wellbore Stress Concentration. The equations for stress around a wellbore shown above (Eqs. 1.8 and 1.9) include terms that describe thermal effects as well as the influence of the internal wellbore pressure. In the latter case, $\Delta P = P_{\text{mud}} - P_{p}$; in other words, the mud acts first against the pressure of the pore fluid, and any excess pressure is then applied to the rock. This assumes a reasonably efficient mud cake, and can be modified to account for its absence. If the mud weight is increased, it results
in an increase in $\sigma_r$ and a decrease in $\sigma_\theta$ and $\sigma_z$; this usually inhibits breakout formation, which explains in part why raising mud weight can often solve wellbore instability problems (see Fig. 1.24). On the other hand, elevated mud pressures increase the likelihood of drilling-induced tensile wall fractures.

Thermal effects at the wellbore wall in the absence of pore fluid diffusion (that is, for purely conductive heat flow) can be described to first order by

$$\sigma^{NT} = (\alpha E \Delta T) / (1 - \nu).$$ ...................................................... (1.13)

There is no effect at the wall of the hole on either $\sigma_r$ or $\sigma_z$. Raising the temperature of the mud leads to an increase in $\sigma_\theta$, which enhances the likelihood of breakouts and inhibits tensile fracture formation; on the other hand, cooling the mud inhibits breakouts (at least as long as the mud is kept at a temperature below the temperature of the rock) and increases the likelihood of development of tensile wall fractures. It has recently been noted that leakoff pressure can be increased by wellbore heating, which is consistent with this effect.

1.5 Determining Stress Orientation

The most reliable way to determine stress orientation is to identify features (either geological features or wellbore failures) the orientation of which is controlled by the orientations of the present-day in-situ stresses. Other methods that rely on observing the effect of stress on rock properties using oriented core have been found to be less reliable and subject to influence by factors other than in-situ stress.

1.5.1 Using Wellbore Failure. As previously discussed, wellbore breakouts occur in vertical wells at the azimuth of $S_{\text{Hmin}}$ and drilling-induced tensile failures occur 90° to breakouts at the azimuth of $S_{\text{Hmax}}$. Therefore, the orientations of these stress-induced wellbore failures uniquely define the orientations of the far-field horizontal stresses when using data from vertical wells.
This is true for breakouts whether they are detected using 4-arm- or 6-arm-oriented caliper logs or using electrical or acoustic images, whether obtained by wireline or LWD tools. In fact, with the advent of density and porosity LWD imaging tools, it is now possible to identify and orient wellbore failures while drilling.

Because mechanical calipers are still the most widely used tool in detecting breakouts, and because of the large amount of available data, a considerable amount of work has been carried out using data from these devices to identify stress orientations and their variations with depth and location. The results, when careful filtering criteria are used, indicate that stress orientations vary slowly with both depth and location. The exceptions are in cases of active faulting, rapid drawdown of compartmentalized reservoirs, and where other local stress perturbations cause changes in the stress field. Figs. 1.25 and 1.26 illustrate that local stress orientations are quite consistent and that stresses generally do not change with depth. In addition, Fig. 1.26 illustrates the expected result that wellbore breakouts and drilling-induced tensile fractures provide similar stress orientation results.
1.5.2 Using Seismic Anisotropy. It has long been known that elastic-wave velocities are a function of stress. That is, velocity will increase with confining pressure, as shown in Fig. 1.27.\textsuperscript{12,13} It has also been demonstrated that this is owing to the presence of microcracks and pores that close in response to applied load. The amount of change of velocity with stress depends both on the number of cracks and on their compliances. As the load increases and the most compliant cracks close, the sensitivity of velocity to confining pressure decreases. Once all cracks are closed, velocities change very little with stress, and under sufficiently high confining stress, it is possible to measure the “intrinsic velocities” of a sample that are functions only of its mineralogy and morphology.

Because rocks are intrinsically anisotropic and can also be anisotropic due to structural fabric such as joints or bedding planes, it is important to differentiate between stress-induced and intrinsic or structural anisotropy. This is rarely possible, except in cases in which the geological structures can be identified and their effects quantified and removed from the data prior to analysis. It is also possible to identify stress-induced anisotropy when the characteristics of the anisotropy that is stress-induced differ from the characteristics of structural or intrinsic anisotropy.

Laboratory experiments have confirmed that, in many rocks, velocities are anisotropic (a function of direction) and are most sensitive to the stress applied in the direction of propagation or of particle motion. Thus, because in-situ stresses vary with direction in the Earth, in-situ velocities are likely to be anisotropic. In the case of compressional $P$-waves, the velocity depends on the direction of propagation because $P$-wave particle motion is parallel to propaga-
tion direction. Under uniaxial stress, the $P$-wave velocity in the direction of applied stress increases with stress much more than the velocity in the direction perpendicular to the applied stress, as shown in Fig. 1.27b. In the case of shear $S$-waves, in which particle motion is perpendicular to propagation direction, velocities depend both on the propagation direction and on the polarization direction (i.e., the direction of particle motion). Because almost all rocks in situ have a finite porosity, this offers the opportunity to derive stress directions from in-situ seismic velocities.

Multicomponent seismic sources and receivers have been developed and deployed, and 3D multicomponent seismic surveys have been designed, to take advantage of this effect. However, while there is clear evidence that vertically and horizontally polarized shear waves have different velocities in shales, it is rare to find an appreciable amount of azimuthal anisotropy. One exception has been in cases in which anisotropy occurs owing to oriented sets of vertical
fractures or joints. In such cases, however, it is not clear if the stress orientations are related in any way to the anisotropy because the joints may have been created in a quite different stress field than pertains at present.

1.5.3 Using Crossed-Dipole Sonic Logs. Modified sonic logging techniques can also be used to determine stress orientations. This is because, in a wellbore, it is possible using oriented sources and receivers to generate modes that bend the borehole in one direction. These dipole modes propagate efficiently at low frequencies and have been used to measure S-wave velocity where standard sonic logs cannot (that is, where S-wave formation velocity is lower than the acoustic velocity of fluids in the well). When velocity is stress-sensitive, dipole velocity becomes a function of the orientations of the sources and receivers, owing both to the presence of a near-wellbore stress concentration and to differences in the far-field stresses. Two modes are produced: a slow shear wave and a fast shear wave. At low frequencies, these propagate at the orientations of the least and greatest stress, respectively, perpendicular to the well. By adding an orientation device to a dipole logging tool, it is possible both to derive the velocities of the fast and slow shear waves and also to determine their directions.

Crossed-dipole logs are recorded in such a way as to allow computation of the velocities and orientations of fast and slow dipole modes. One potential benefit of these analyses is that theoretical considerations and laboratory measurements indicate that it is possible to differentiate between stress-induced and intrinsic anisotropy based on plots of velocity vs. frequency of the dipole modes. However, it is rare that field data are of sufficient quality to make this possible. In fact, reliable stress orientations are generally only possible in sands, porous limestones, and shales where the well is nearly perpendicular to bedding, where intrinsic anisotropy is low and the rocks are fairly compliant. In addition, these measurements should only be attempted when wells are drilled within approximately 20° of a principal stress direction. Finally, analyses should be believed only when very restrictive quality-assurance conditions can be met (e.g., Fig. 1.28).

1.5.4 Core-Based Analysis of Stress Orientation. Considerable effort has been devoted to developing and validating core-based stress analysis techniques. The one thing these have in common is the idea that post-coring deformation is dominated by expansion occurring because of removal of the core from in-situ stress conditions. The assumption is that the recovery-induced strains will have the same relative magnitudes and orientations as the original in-situ stresses. Therefore, by measuring the strains caused by removal or reloading, it is possible to constrain at least the directions of the principal stresses and their relative magnitudes.

There are basically three classes of techniques. These include measuring strain relaxation as a function of time after core removal, measuring strain as a function of orientation while reloading under isotropic conditions (possibly including monitoring for noise caused by microscopic slip events), and measuring velocities as a function of orientation under isotropic reloading. To orient stresses based on these techniques, it is necessary to know the original orientation of the core, which adds to the complexity of coring operations. Also, because the orientations of the principal strains are unknown prior to testing, to determine them it is necessary to attach more than three strain gauges to the sample.

In strain relaxation measurements, the core is recovered as quickly as possible, instrumented with strain gauges to monitor deformation as a function of time, and maintained in a constant (fixed) temperature/humidity environment. The principal strain axes are assumed to define the principal in-situ stress axes, and the relative strain magnitudes are assumed to correspond to the relative stress magnitudes. Thus, the vertical and horizontal relative stress magnitudes and the horizontal stress orientations can be derived from the principal strain orientations and magnitudes. Because most of the strain occurs during the first few minutes following removal from in-situ conditions, rapid recovery is essential to ensure accurate results.
The second (reloading) technique relies on the assumption that samples are much “softer” at stresses that are below their original confining stress than they are at stresses above their original confining stress. This assumption can be extended to cases in which the original stress state was anisotropic, so that the stress at which the sample gets stiffer is different in different directions. By isotropically loading a rock and monitoring strain as a function of confining stress and orientation, it is possible to determine the magnitudes of the three principal stresses by identifying the point in plots of stress vs. strain at which each curve bends over, indicating that the sample has suddenly become less sensitive to applied stress. The in-situ stress orientations and Andersonian stress state can be derived using the relative stress magnitudes at which this occurs, after resolving them into principal stress coordinates. If the sample has been instrumented to observe acoustic emissions from microscopic slip events, these will sometimes increase once the in-situ stress has been exceeded.

To derive principal stress orientations using velocity measurements, samples are instrumented with ultrasonic transmitters and receivers at a number of orientations, and the travel times of ultrasonic pulses are measured as a function of confining stress. As in the case of reloading, changes in velocity for each principal stress direction while confining pressure is below the original stress are larger than changes in velocity when confining pressure is above the original stress.

All three techniques suffer from the same limitation, which is that nearly all rocks are intrinsically anisotropic. In other words, their elastic moduli (which control the amount of strain that is caused by a given applied stress) are a function of direction. Anisotropic rocks will have
different amounts of strain in different directions, even if they are subjected to an isotropic stress state. If the intrinsic anisotropy is large enough, which it generally is in shales, laminated sands, and other finely bedded or foliated rocks, strains related to that anisotropy can mask strains caused by stress changes. Thus, while there are some situations in which these techniques work, there are many pitfalls, and the results should be used with caution.

1.5.5 Geological Indicators of Stress Orientation. In the absence of better data, it is sometimes useful to look at earthquake focal mechanisms within the region or to map local geological structures to help provide a “first look” estimate of the relative magnitudes and orientations of the current stresses. In the case of earthquake focal mechanisms, it is important to utilize data from many earthquakes within a small region to derive a “composite focal mechanism,” to avoid the large uncertainties associated with individual analyses. In the case of geological structure, it is critically important to remember that many structures are inherited from older stress fields and that the only structures that do provide information are those that are currently active. Fig. 1.29 is an example, from South Eugene Island in the Gulf of Mexico, where the stress direction, confirmed by wellbore breakout analysis, is consistent with the orientation of a nearby large, active normal fault.

Salt domes can significantly perturb the local stress field because extension predominates above active salt intrusions, whereas beside the salt compression acts radially away from its walls. This is because salt is virtually unable to sustain a significant stress difference, and thus all three stresses in salt bodies are nearly equal and close to the vertical stress. This not only increases the local horizontal stresses, but also causes a rotation in the principal stress axes to be perpendicular and parallel to the salt face. Salt domes rarely have vertical walls, and thus the vertical stress may no longer be a principal stress close to their flanks.

Fig. 1.29—Structure map in the South Eugene Island area of the Gulf of Mexico, showing large, active, WNW-ESE-trending normal faults. Given this fault orientation, the least stress is expected to be horizontal and oriented NE-SW. Stress orientations from breakouts confirm this (outward-facing arrows) (modified after Finkbeiner et al.)
1.6 Building the Geomechanical Model

The elements of the geomechanical model that form the basis for analysis of wellbore stability are the state of stress (the orientations and magnitudes of the three principal stresses), the pore pressure, and the rock properties, including strength (which can be anisotropic, particularly in consolidated shales). We have already presented a number of ways to determine the rock strength and the orientations of the in-situ stresses. In this section, we outline methods for determination of the stress magnitudes and the pore pressure.

1.6.1 Overburden, $S_v$. Overburden pressure, or $S_v$, is almost always equal to the weight of overlying fluids and rock. Thus, it can be calculated by integrating the density of the materials overlying the depth of interest (see Fig. 1.30).

$$S_v(Z_o) = \int_0^{Z_o} \rho_b G d z. \quad \text{......................................................... (1.14)}$$

Here, $G$ is the gravitational coefficient. The best measurement of density is derived from well logs. However, density logs are seldom acquired to ground surface or to the sea floor. If good seismic velocities are available, a velocity to density transformation can be used to estimate density where it has not been measured directly. A number of transformations from velocity to density are available (see Eq. 1.15 and Table 1.3).

In the absence of good velocity or density data, densities must be extrapolated from the surface to the depths at which they are measured. Shallow density profiles can take many forms, and thus they ideally should be calibrated against in-situ log data or measurements of sample densities. A good resource for information about shallow density profiles is the archives of the Deep Sea Drilling Project and the Ocean Drilling Program (www.oceandrilling.org). In the absence of good calibrations or data, reasonable mudline densities for clean sands are between 2.0 and 2.2 gm/cm$^3$, and for fine-grained shales are between 1.4 and 1.8 gm/cm$^3$.

$$\rho_b = a V_p^b,$$

and

$$\rho_b = c V_p^2 + d V_p + e, \quad \text{......................................................... (1.15)}$$

where $a$, $b$, $c$, $d$, and $e$ are constants that vary with lithology. The values in Table 1.3 are for velocity in km/s and density in gm/cm$^3$.

1.6.2 Pore Pressure, $P_p$. The only accurate way to determine pore pressure is by direct measurement. Such measurements are typically done in reservoirs at the same time fluid samples are taken with a wireline formation-testing tool. Recently, advances in while-drilling measurements make it possible to measure in-situ pore pressure while drilling. However, it is difficult
(if not impossible) to measure pore pressure in shales because of their very low permeability and small pore volume. In addition, because of their low permeability, pore pressure in shales adjacent to permeable reservoirs may be different from pore pressures in the reservoir. However, there are a number of methods that can be used to estimate pore pressure in shales based on other measurements. Because pore pressure (and the derived fracture gradient, which will not be discussed) is often the only geomechanical parameter on which mud weights are based, we will take some time to review standard and new methods for its prediction. Keep in mind that these prediction methods are intended for use only in shales.

Pore-pressure-prediction methods fall into a few general categories. In the first category are normal compaction trend (NCT), ratio, and equivalent depth methods, which are all more or less empirical. The second category includes methods that explicitly utilize relationships between measured values and the effective stress. These first two methods assume that the in-situ material is either normally compacted or undercompacted. In the third category are models that are also applicable to overcompacted rock. All of these methods require measurement of one or more physical properties that are functions of effective stress. These include resistivity, density, and seismic or sonic velocity.

In most cases, the only measurement that is available prior to drilling is (P-wave) seismic velocity. After the first well has been drilled, or during drilling (using LWD tools), log data are acquired that make it possible to improve predrill pore-pressure estimates. Using LWD, and adding a pressure while drilling (PWD) measurement, pore-pressure analysis can be carried out in real time. Typically, in shale sections above a target, only LWD resistivity and gamma ray are acquired, but deeper in the well, additional measurements are often made, including density and velocity.

One additional measurement that has been used to predict pore pressure is the drilling exponent, $D_c$, which defines the rate of drill-bit penetration as a function of depth. Because ease of
drilling is related to strength, which in turn is a function of porosity (and therefore of effective stress), the rate of penetration should be a function of effective stress, provided that it is corrected for changes in any other drilling parameters. Therefore, \( D_c \) can be used to determine pore pressure using the same analyses used to compute pore pressure from physical properties like resistivity or velocity.

Although all shale pore-pressure-prediction methods rely on the fact that rock physical properties depend on the effective stress, \( \sigma = (S - \alpha P_p) \), equivalent depth and NCT methods use depth as a proxy, and in ratio methods even depth is implicit. Effective stress methods work by

1. Measuring the total stress \( S \).
2. Using either an explicit relationship or an implicit function to derive the effective stress \( \sigma \) from a measured parameter.
3. Computing the pore pressure as the difference between the effective stress and the total stress, divided by \( \alpha \), the Biot coefficient \( [P_p = (S - \sigma)/\alpha] \).

In relatively young, unconsolidated shales \( \alpha = 1 \), but values of 0.9 or less may be more appropriate for older, more highly compacted sediments.

Because overburden \( S_v \) can be computed as the integral of the density of the rock and fluid overlying the depth of interest, pore-pressure-prediction methods were developed initially using \( P_p = (S_v - \sigma) \). This is more reasonable than it might seem because properties such as velocity depend most strongly on stress in the direction of propagation, which for near-offset seismic data is nearly vertical. In some cases, the vertical stress is replaced by the mean stress. This results in adjustments to the pore pressure computations based on differences in the magnitudes of the horizontal stresses in different regions.

The relationship between the measured quantity and the effective stress is derived either using explicit functional relationships or by so-called trend-line methods. Trend-line methods require the existence of a depth section, over which the pore pressure is hydrostatic, to derive the NCT.

**Equivalent Depth Methods.** One example of analysis using a trend line is the equivalent depth method illustrated in Fig. 1.31. This method first assumes that there is a depth section over which the pore pressure is hydrostatic, and the sediments are normally compacted because of the systematic increase in effective stress with depth. When the log of a measured value is plotted as a function of depth, NCTs can be displayed as straight lines fitted to the data over the normally compacted interval. Because the value of the measured physical property is a unique function of effective stress, the pore pressure at any depth where the measured value is not on the NCT can be computed from

\[
P_z = P_a + (S_z - S_a), \quad \text{........................................................} \quad (1.16)
\]

where \( P_{a,z} \) and \( S_{a,z} \) are the pore pressure and the stress at \( z \), the depth of interest and \( a \), the depth along the normal compaction trend at which the measured parameter is the same as it is at the depth of interest. The only unique assumption required by equivalent depth methods is that effective stress is a linear function of depth.

**The Ratio Method.** In the ratio method, pore pressure is calculated using the assumption that, for sonic delta-t, density, and resistivity, respectively, the pore pressure is the product of the normal pressure multiplied (or divided by) the ratio of the measured value to the normal value for the same depth.

\[
P_p = P_{\text{hyd}} \frac{\Delta T_{\log}}{\Delta T_n},
\]

\[
P_p = P_{\text{hyd}} \frac{\rho_n}{\rho_{\log}},
\]

and
where the subscripts \( n \) and \( \text{log} \) refer to the normal and measured values of density, resistivity, or sonic delta-t; \( P_p \) is the actual pore pressure, and \( P_{\text{hyd}} \) is the normal hydrostatic pore pressure. Calibration of this method requires knowing the appropriate normal value of each parameter. It is important to realize that, in contrast to trend-line methods, the ratio method does not use overburden or effective stress explicitly and thus is not an effective stress method. This can lead to unphysical situations, such as calculated pore pressures that are higher than the overburden. The ratio method is also applied to analyses of pore pressure from the drilling exponent (Fig. 1.32).\(^{16}\)

**Eaton’s Method.** Perhaps the most widely publicized pore-pressure-estimation technique is Eaton’s method, shown graphically in Fig. 1.33.\(^{16}\) Here, stress is used explicitly in the equations

\[
P_p = S - (S - P_{\text{hyd}})(R_{\text{log}} / R_n)^{1.2}
\]

and

\[
P_p = S - (S - P_{\text{hyd}})(\Delta T_n / \Delta T_{\text{log}})^{3.0}, \text{ ........................................... (1.18)}
\]
where $P_p$ is pore pressure; $S$ is the stress (typically, $S_v$); $P_{hyd}$ is hydrostatic pore pressure; and the subscripts $n$ and $\log$ refer to the normal and measured values of resistivity ($R$) and sonic delta-t ($\Delta T$) at each depth. The exponents shown in Eq. 1.18 are typical values that are often changed for different regions so that the predictions better match pore pressures inferred from other data.

The major problem with all trend-line methods is that the user must pick the correct normal compaction trend. Sometimes are too few data to define the NCT. Unfortunately, if the NCT is defined over an interval with elevated pore pressure, the method will give the wrong (too low) pore pressure, leading to severe risks for drilling.

**Effective Stress Methods.** Methods that treat the problem correctly are often referred to as effective stress methods. The basis of the approach is summarized in Fig. 1.34. In this example, the top set of plots shows data recorded over a normally compacted section. The mean stress and pore pressure are shown as a function of depth at the left. Because the effective stress increases with depth, the porosity decreases with depth, as shown in the middle. If the porosity-stress function is a power law, it will plot as a straight line in linear-log space. There is, in fact, no restriction on the functional form of the porosity-stress function. The lower set of plots in Fig. 1.34 show the effect of an increase in pore pressure below a certain depth, as represented by the dashed line that diverges from the hydrostatic line in the lower left plot. In this case, the pore pressure in the overpressured zone increases at the same rate as the mean stress, such that the effective stress is constant. The plot of log $\sigma$ vs. $\Phi$ follows the compaction trend only until it reaches the depth of overpressure, after which there is no change in either porosity or effective stress (lower right plot). This type of profile is typical of regions in which a pore-pressure increase is caused by the inability of the pore fluids to escape during burial and compaction.

In general, effective stress methods must be calibrated, preferably using log data. However, they can also be calibrated empirically using approaches similar to those used to select trend lines, and they account explicitly for local changes in overburden and other stresses. The equation plotted in Fig. 1.34 is an example of relationships of the form

![Fig. 1.32—Illustration of the ratio method. Here, $d_n$ is the expected value of the drilling exponent based on extrapolating a normal trend, and $d_{co}$ is its measured value (modified after Mouchet and Mitchell).]
\[ \sigma = \sigma_o e^{-\beta \nu} \]  \hspace{1cm} (1.19)

Athy\(^{18}\) first proposed this type of relationship in 1930, also proposing values for the parameters. Use of Athy’s original parameters is not recommended because they were based on analysis of overconsolidated shales from Oklahoma and thus are not applicable to young sediments. An appropriate algorithm for Athy’s method is to solve the following set of equations.

\[ P_p = S_v - \left[ \frac{1}{\beta} \ln \left( \frac{\Phi_o}{\Phi} \right) \right], \]

where

\[ \Phi = 1 - \left( \frac{\Delta t_{ma}}{\Delta t} \right)^{1/f}, \]  \hspace{1cm} (1.20)

and \( f \) is the acoustic formation factor and is derived by calibration; \( \Delta t_{ma} \) is the matrix transit time. This type of relationship allows extension to account for effects such as cementation and thermal transformations by modifying the functional form of the exponent.

**Complications.** Ultimately, all pore-pressure methods must be calibrated; this is done empirically in most cases. Typical approaches rely on drilling experience to provide calibration points. These calibration points are based either on the occurrence of kicks, in which case the pore pressure in the sand producing the kick must be higher than the equivalent mud weight and lower than the kill mud weight, or on observations of instabilities in shales. In the former case, it is assumed that the pore pressures in shales and the adjacent sands are the same. In the latter case, the assumption is that instabilities occur when the mud weight has fallen below the pore pressure. In fact, wellbore instabilities that are due to compressive breakouts can occur at pressures that are higher or lower than the pore pressure. Therefore, the assumption that collapse begins to occur at a mud weight equal to the pore pressure can result either in an

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Fig. 1.33—Lines for computing pore pressure expressed as an equivalent density, calculated using Eaton’s method and the drilling exponent. Notice that these “lines” are not linear in semilog space (modified after Mouchet and Mitchell\(^{16}\)).
overestimate or an underestimate of $P_p$. If neither occurs, $P_p$ is assumed (sometimes incorrectly) to be less than $P_{mud}$.

A further complication is that all of these methods require that the rock obeys a single, monotonic, compaction-induced trend, and that no other effects are operating. In reality, active chemical processes can increase cementation, leading to increased stiffness (higher velocities), which can mask high pore pressure, and increased resistance to further compaction, which can lead to erroneous prediction of the onset of pore pressure. Elevated temperatures lead to a transformation of the predominant shale mineral. For example, an increase in temperature transforms a water-bearing smectite to a relatively water-free (and more dense) illite. This transformation occurs over a range of temperatures near 110°C, but they can vary with fluid chemistry; furthermore, the depth at which this temperature is exceeded varies from basin to basin. Dutta developed a method that expands the argument of the exponential relationship of Eq. 1.19 to account for temperature effects and diagenesis (cementation and other changes that occur over time).

Pore fluid properties can also have a significant effect on pore-pressure predictions. This is because resistivity and velocity are both affected by the type and properties of the pore fluid. Changes in the salinity of brines will change resistivity, because pore fluid conductivity increases with salinity; thus a salinity increase (for example, adjacent to or beneath a salt dome) could be misinterpreted as an increase in pore pressure. Fluid conductivity is also a function of temperature.

Substitution of hydrocarbons for brine will increase resistivity, because hydrocarbons do not conduct electricity; this can mask increases in pore pressure that often accompany the presence of hydrocarbons.

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Fig. 1.34—Illustration of the effective stress method for pore pressure prediction (modified after Swarbrick).
of hydrocarbons. Because hydrocarbons are more compliant and less dense than brines, compression-wave velocity will decrease and shear-wave velocity will increase as hydrocarbon saturation increases. High gas saturation or API index will amplify this affect. Because a change from water to hydrocarbon affects resistivity and compressional velocity in opposite ways, simultaneous pore-pressure analyses using both measurements can sometimes identify such zones. It is more difficult to identify and deal with changes in fluid salinity.

**Undercompaction.** Most shale properties are, fortunately, characterized by fairly simple and single-valued functions of effective stress while on the compaction trend. When unloading occurs and the material becomes overcompacted, they do not follow the same relationship. This is because when the effective stress decreases, porosity and other properties are less sensitive to effective stress (see Fig. 1.11). Fortunately, relationships between porosity (or density) and other properties are different for overcompacted sediments than they are for the same sediment when it is normally compacted or undercompacted, as shown in laboratory data (Fig. 1.35). This provides a way to differentiate between undercompacted and overcompacted shales, using plots of velocity vs. density (Fig. 1.36). Once the domains have been separated, independent calibrations can be used to determine the pore pressure.

In highly lithified, older sediments, as in the case of overcompacted sediments, it is very difficult to use trend-line analyses to determine pore pressure. This is because, in these sediments, the sensitivity of porosity to effective stress is small. Even in such cases, however, it is sometimes possible (with accurate models derived from laboratory measurements and calibrated against in-situ direct measurements) to utilize resistivity or velocity measurements to estimate pore pressure.

**Centroid and Buoyancy Effects.** The previous discussion of pore-pressure-prediction algorithms applies exclusively to shales and other low-permeability materials that undergo large amounts of shear-enhanced compaction. Because these algorithms do not work very well in sands, it is often assumed that pore pressures in sands are similar to those in adjacent shales. In reality, this is often not the case because the low permeability of shale makes it possible for it to maintain a pore pressure that is quite different from that in the adjacent sand. Two active processes, both of which can lead to very much higher (or lower) sand pore pressure than shale pore pressure that can be maintained over geological time because of low shale permeability, are the centroid and buoyancy effects.

The classical centroid effect occurs when an initially flat reservoir surrounded by and in equilibrium with overpressured shale is loaded asymmetrically and tilted, leading to a hydrostatic gradient in the sand that is in equilibrium with the original pore pressure at the depth of the sand prior to tilting. At the same time, pore pressure in the shale, which has extremely low permeability after it has been compacted, changes in such a way as to maintain a constant effective stress equal to the original effective stress at the depth of the sand prior to tilting. At the depth of the centroid (usually taken to be the mean elevation of the sand), the shale and sand pore pressures are equal. This effect is shown diagrammatically in Fig. 1.37. Because the pressure in the shale decreases upward at the same rate as the overburden (that is, proportional to the density of the shale itself), it is much lower at the top of the reservoir than is the pore pressure within the reservoir, which decreases at a slower rate that is proportional to the fluid density in the reservoir. Below the centroid, pore pressure in the sand is less than that in the adjacent shale.

Buoyancy effects occur when hydrocarbons begin to fill a tilted reservoir. The lighter hydrocarbons migrate to the top of the structure. Pressure at depth within the reservoir still follows a hydrostatic gradient (Fig. 1.38). The pressure in the gas at the top of the reservoir decreases upward more slowly, at a rate proportional to the density of the gas, which can be less than one-fourth the density of water. This leads to elevated pressure at the top of the structure. The
same process occurs when oil fills a reservoir, but since the density difference is not as large for oil, the effect is less pronounced.

The reservoir can continue to fill until the structure’s sealing capacity is exceeded. In the example, seal capacity is exceeded when the pressure at the top of the reservoir is high enough to cause the sealing fault to slip. However, in extreme cases, the reservoir pressure can be close to the least principal stress in the adjacent shale.

1.6.3 Least Principal Stress, \( S_3 \). The least principal stress can be measured directly, using either extended leakoff tests or minifrac tests. These tests are similar to casing integrity tests or standard leakoff tests, except that the test procedure is slightly modified. Fluid is pumped into the wellbore to pressurize a short interval of exposed rock until the rock fractures and the frac-
ture is propagated a short distance away from the well by continued pumping. In either case, pumping is carried out at a constant rate, and pressure and the volume of fluid pumped are recorded as a function of time. Pressure-time curves typically look like those in Fig. 1.39.

The theory behind using these tests to measure $S_3$ is that a fracture created during the test will, to minimize the energy required for its propagation, grow away from the well in an orientation that is perpendicular to the far-field least principal stress. Therefore, the pressure required to propagate the fracture will be equal to or higher than the least stress. Fracture propagation will stop when leakoff of fluid from the fracture and wellbore and into the formation occurs faster than the fluid is replaced by pumping. If pumping stops entirely, fluid leakoff will continue from the walls of the fracture until it closes, severing its connection to the wellbore. The fracture will close as soon as the pressure drops below the stress acting normal to the fracture (which is the least principal stress). The change in flow regime after pumping stops, from one in which the fracture contributes to fluid losses to one in which all fluid losses occur through the walls of the well, can be seen in pressure-time and other plots of pressure after shut-in (for example, pressure vs. the square root of time, Fig. 1.40). The least principal stress is taken to be the pressure at which the transition in flow regime occurs. This pressure is a clear indication of the least stress regardless of whether the test created one or a multitude of subparallel fractures, as modeling suggests sometimes occurs.

Recently, and with evidence based on the ability to control pressure flowback in microfrac tests, it has been suggested that fracture closure can overestimate the least principal stress, and that choked flowback at various rates is required to determine that stress accurately. However, until such techniques become available for use in leakoff tests, the approach outlined below, based on the above theory, is the best for measuring $S_3$ in practice.

As for casing integrity tests or standard leakoff tests, extended leakoff tests are conducted after cementing a casing string and drilling out a short section (often between 20 and 50 ft) below the casing shoe. The conduct of the test should be as follows (refer to Fig. 1.39):

1. Perform a pretest pumping cycle with the formation isolated from flow using a very low flow rate. Pump until the pressure reaches a predefined upper limit for the casing. Record pressure and flow rate as a function of time, and draw a pressure-volume curve. This gives you a plot of pressure vs. volume (the slope of which is the system stiffness) if no fracture is initiated in the formation. The pressure vs. volume plot may initially be slightly concave up,
indicating that the mud is compliant, possibly owing to entrained gas that is being forced into solution. Alternatively, refer to tables for the particular wellbore fluid and plot the appropriate pressure vs. volume curve by hand.

2. Open the formation to the well, pump at a low rate (¼ to ½ bbl/min), and overlay a plot of pressure vs. volume pumped on the curve from the casing test. The initial inflation should be approximately parallel to (or a little less steep than) the casing test curve. A concave down curve may be an indication of losses into the formation or a shoe integrity problem. If the former, it is possible to overcome this problem by stopping, flowing back, and starting again with a higher flow rate. If the latter, the problem must be dealt with before proceeding.

3. Pump until one of two things happens: either you have pumped a fixed volume of fluid above what was required to reach a given pressure (in which case there is a fluid loss problem to be dealt with), or the inflation pressure curve will break over, indicating the creation of a hydraulic fracture.

4. In a leakoff test, the formation would be shut in as soon as the slope of the pressure vs. volume curve begins to flatten, which is defined as the leakoff point. The fracture gradient at the shoe would be set equal to the pressure at the leakoff point. This is (in general) an upper bound on the least stress, and can, in the absence of better data, be used with caution in geomechanical models. However, to determine the least stress more accurately, continue pumping until the pressure stabilizes or begins to drop, and then shut in by stopping the pumps.

5. Record the pressure after shut-in until the pressure stabilizes. The value to which the pressure drops immediately after the pumps are shut off is typically called the instantaneous shut-in pressure (ISIP).

6. The least stress is determined using a variety of analysis methods. One method that is commonly used is to plot pressure vs. the square root of time after shut-in. The fracture closure pressure is defined by a change in the curvature of the line, as shown in Fig. 1.40.23

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**Fig. 1.37**—This figure shows diagrammatically a typical centroid geometry (left) and pore pressure profiles (right) in a reservoir sand and in the surrounding shale that develop because of the centroid effect. Pressure at the top of the sand is higher than in the adjacent shale, whereas pressure at the base of the sand is lower (modified after Bruce and Bowers21).
7. Ideally, a second cycle should be performed. In this cycle, the previously created fracture is re-opened and extended, and then shut in again. Either or both of the following “step-rate tests” can be employed to refine the least principal stress measurement (Fig. 1.41).

(a) Re-open the fracture, starting with very low flow rates, and increasing the flow rate in discrete steps until the fracture opens and starts to grow. Maintain a fixed flow rate at each step until the pressure equilibrates. A plot of pressure vs. flow rate will have two slopes. At low pressure, fluid losses into the formation will result in a radial flow pattern in which flow rate increases systematically with pressure. Once the fracture is open, fracture growth and losses from the fracture walls will cause the pressure to increase much more slowly with flow rate. The intersection of lines fit to these two trends provides an upper bound on $S_3$. For a viscous fluid, extrapolation of the latter fit to zero flow gives a lower bound. The benefit of this procedure over 7b is that there is no need to extend the fracture as far.

(b) Open the fracture, pumping at the same rate as in the first cycle. The pressure at which a plot of pressure vs. volume deviates from the first cycle is the fracture reopening pressure. The difference between that pressure and the leakoff pressure is a measure of the formation tensile strength. Once the fracture begins to extend, decrease the pump rate in fixed increments, recording flow rate and pressure after the pressure has equilibrated at each step. Analyze this data using the same technique as described for step-rate reopening. The benefit of this procedure over 7a is that a measure of tensile strength can be obtained.

1.6.4 Estimates of Least Principal Stress From Ballooning. Ballooning is a process that occurs when wells are drilled with equivalent static mud weights close to the leakoff pressure. It occurs because during drilling, the dynamic mud weight exceeds the leakoff pressure, leading to near-wellbore fracturing and seepage loss of small volumes of drilling fluid while the pumps are on. When the pumps are turned off, the pressure drops below the leakoff pressure, and the fluid is returned to the well as the fractures close. This process has been called “breathing” or “ballooning” because it looks like the well is expanding while circulating, and contracting once the pumps are turned off. This behavior can be identified on a PWD log (Fig. 1.42a).

Fig. 1.38—This figure illustrates the buoyancy effect caused by systematic filling of a reservoir with low-density hydrocarbons. As filling progresses from Stage 1 to 2 to 3, the gas column grows, but the pressure is always in equilibrium with the centroid pressure at the gas-water contact, and so the pressure at the top of the reservoir increases (courtesy GeoMechanics Intl. Inc.).
crease mud weight owing to the perception that it reveals gas pressures higher than the equivalent static mud weight), as shown in Fig. 1.42b. Increasing the mud weight in a ballooning well can lead to massive lost circulation.

Balooning is an important measure of least principal stress magnitude because it is essentially an inadvertent leakoff test conducted while drilling. The static mud weight is a lower bound on the magnitude of the least stress, and the dynamic mud weight is an upper bound. In some cases, a shut-in break can be detected, which is a very accurate measure of the least stress. The only problem is that it can be difficult to identify the depth at which the ballooning incident took place (although it is reasonable to assume that it occurred close to the bit). This is a particular problem when there is a very long openhole interval. Fortunately, it is often
possible to find the location of the fractures created by the ballooning incident by a change in LWD resistivity recorded before and after the event.

1.6.5 Using Wellbore Failure to Constrain the Magnitude of $S_{H_{\text{max}}}$. Once independent knowledge of $S_v$ and $S_{H_{\text{min}}}$ is available, $S_{H_{\text{max}}}$ can be determined from the widths of wellbore breakouts in vertical boreholes. Because the stress concentration around the well and the rock strength are equal at the point of the maximum breakout width, it is possible to re-arrange Eq. 1.8 to solve for $S_{H_{\text{max}}}$, as shown in Fig. 1.43. Solving for $S_{H_{\text{max}}}$ also requires a model for rock strength and knowledge of the pore pressure and mud weight. While the equations presented here are technically accurate only for elastic, brittle rock, utilizing the results to select the appropriate mud weight for drilling future wells requires only that the same model be applied to predict wellbore stability as was used to determine the stresses.

Once breakouts have formed, they deepen but do not widen. Thus, the original width of the breakout is largely preserved, and calculations of stress magnitudes based on breakout width do not have to be adjusted for changes in the wellbore shape associated with subsequent failure (see Fig. 1.44). $^{8,25}$

As previously discussed, breakout width can be determined very accurately using acoustic or electrical image logs run after the well has been drilled. With the advent of resistivity, density, and porosity LWD tools that produce an image of the borehole wall behind the bit, it is...
now possible to determine breakout widths while drilling, which then makes it possible to determine $S_{H_{\text{max}}}$ in real time. On the other hand, in the absence of borehole image data, we can only place bounds on the width of presumed breakouts if they can be detected using the electrode pads of a dipmeter tool (pad width is typically about 30° in an 8.5-in. hole). Therefore, using mechanical calipers, it is possible only to place constraints on the magnitude of $S_{H_{\text{max}}}$.

The presence of tensile fractures in a well also gives some indication of relative stress magnitudes. This is because, as previously discussed, tensile fractures can develop at the wellbore wall only if the far-field horizontal stresses are sufficiently different. For example, when the mud weight is equal to the pore pressure, a strike-slip equilibrium state of horizontal stress is required for tensile wall fractures to develop in a vertical well.

**Constraining the Magnitude of $S_{H_{\text{max}}}$ in Deviated Wellbores.** In deviated wellbores, it is possible to constrain not only the orientation but also the magnitude of $S_{H_{\text{max}}}$. This is because,
in deviated wells, the position of wellbore breakouts depends on stress magnitude as well as on stress orientation (Fig. 1.45). It is also possible at the same time to constrain the rock strength using breakout width.

This sort of analysis can be carried out in multiple wells by use of combined analyses of tensile and compressive wellbore failures. If the wells have a sufficient number of different deviations and azimuths, a very accurate stress state can be determined using a Monte Carlo approach. Essentially, this is simply a more quantitative way of doing the same thing as creating a figure similar to Fig. 1.45 for each of the wells and overlaying the figures to identify the one stress state that comes closest to matching all of the observations. If the results for all wells are not consistent with a single stress state, then it is clear that the stress state must be different at the locations of the anomalous wells. This provides powerful evidence for reservoir compartmentalization or the influence of local sources of stress.

The constraints on in-situ stress dictated by the strength of pre-existing faults shown in Fig. 1.10 can be combined with observations of tensile and compressive wellbore failures to refine estimates of in-situ stress, as shown in Fig. 1.46. The frictional strength limits are as described above. Overlaid on these limits are lines defining the stress states for one specific well that would cause tensile or compressive failure to occur. The near-vertical, fine lines to the left of the stress polygon represent stress states (values of \( S_{\text{Hmax}} \) and \( S_{\text{Min}} \)) that would create tensile fractures at the wall of this deviated well for tensile strengths of 0, 500, and 1,000 psi. If the rock has a given tensile strength and tensile cracks are found, it indicates that the stress state must lie to the left of the appropriate line. Because in most cases pre-existing flaws exist that can be opened by elevated mud weights, the effective tensile strength is often assumed to be zero. Therefore, for this example, it is apparent that if tensile failure is observed, the stress state must lie at the extreme lower left-hand corner of the strike-slip region or the extreme left side of the normal faulting region, a transitional strike-slip or normal faulting stress state for

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**Fig. 1.43**—Schematic diagram of a breakout and the Kirsch equations that are used to constrain stress magnitudes based on the widths of wellbore breakouts and the presence or absence of drilling induced tensile wall fractures. These equations apply to a vertical well when \( S_v \) is a principal stress (courtesy GeoMechanics Intl. Inc.).
which \( S_{H_{\text{max}}} \) can range from 14 to as high as 38 lbm/gal. \( S_{H_{\text{min}}} \) is much better constrained to between approximately 13 and 15 lbm/gal.

On the other hand, if no tensile fractures are observed and lab or log data indicate that \( C_o \) is between 10,000 and 15,000 psi, the stress state can lie anywhere to the right of the vertical lines, and within the region between the near-horizontal, light gray curves plotted for those values of \( C_o \). In other words, \( S_{H_{\text{min}}} \) can range from 13 to as high as 30 lbm/gal, and \( S_{H_{\text{max}}} \) is somewhat better constrained to a range from 26 to 33 lbm/gal. If \( S_{H_{\text{min}}} \) had been measured using an extended leakoff test to be approximately 20 lbm/gal, then the range of possible \( S_{H_{\text{max}}} \) values would be only slightly smaller (between 28 and 33 lbm/gal). In general, in near-vertical wells, the presence of tensile cracks severely limits the magnitude of \( S_{H_{\text{min}}} \) without constraining \( S_{H_{\text{max}}} \). Multiple observations of breakouts provide weaker constraints on \( S_{H_{\text{min}}} \) than on \( S_{H_{\text{max}}} \).

When using this sort of analysis, the important thing to keep in mind is that all you are doing is providing constraints on the stress state. For example, suppose that no breakouts had formed in the well described by Fig. 1.46 and the rock strength was somewhere between 10,000 and 15,000 psi. In that case, the stress state could definitely not lie above the line corresponding to \( C_o = 15,000 \) psi, and is most likely to lie below the line corresponding to \( C_o = 10,000 \) psi (i.e., anywhere within the low-stress region, which includes the entire normal faulting stress regime). Additional observations would be required to reduce the large uncertainty in this result.

**Constraining the Stress State in the Visund Field.** As an example of an instance in which redundant data confirm the stress and strength values derived from combined analysis of wellbore failure and frictional constraints, consider Fig. 1.47,26 prepared on the basis of data from an inclined well in the Visund field, North Sea. The frictional faulting constraints were derived from \( S_v \) and \( P_p \) calculated as described above. Breakouts were identified in caliper data, and intermittent tensile fractures were also seen in both vertical and inclined sections of the well.
Breakouts and tensile cracks in the vertical section provided information on the stress orientation. Based on log data, \( C_0 \) ranged from 20 to 25 MPa. The light gray lines labeled 20 and 25 correspond to the stresses constrained by the breakout observations and the rock strength parameters.

Because tensile cracks are more likely to occur when circulation cools the well, it is necessary to account for that cooling in the stress constraints from their occurrence. That shifts the tensile failure line to the right. It is not necessary to include cooling in the breakout analysis, however, because the breakouts would be more likely to occur after the well temperature had equilibrated. The final constraint, based on frictional faulting theory, is that the stress state cannot lie outside the polygon.

Taken together, these observations constrain the stress state to lie in the small region bounded by the light gray lines on the top and bottom, the thin dark near-vertical line on the right, and the edge of the stress polygon on the left. This provides a very precise value for \( S_{H_{\text{min}}} \) between 52.5 and 54.5 MPa, and it constrains \( S_{H_{\text{max}}} \) to be between 73 and 76 MPa. A leakoff
test provided redundant information on $S_{\text{Hmin}}$ and confirmed its value predicted from the constraints imposed by observations of failure.

1.7 Predicting Wellbore Stability

Once a geomechanical model has been developed that quantifies the principal stress magnitudes and orientations, the pore pressure, and the rock properties, it is possible to predict the amount of wellbore instability as a function of mud weight and properties. This makes it possible to reduce drilling costs by keeping lost time low and by designing wells just carefully enough to minimize problems without excessive cost. A further benefit of considering geomechanical risk is that when problems are encountered, their causes can be recognized and plans can be in place to mitigate their effects with minimal disruption of the drilling schedule.

Fig. 1.48 shows the time-depth plot of an offshore well that was designed and drilled without the use of geomechanical modeling. After setting the first string to isolate a shallow hazard, the remaining casing depth points were selected based on drilling experience in an offset block, supplemented by pore pressures predicted using seismic data. Considerable problems were experienced because of the length of the fourth casing interval. Subsequent geomechanical analysis revealed that the fourth casing interval was too long because the second and third casing strings were set too shallow (the dark dashed line on Fig. 1.48 representing AFE). A
A new casing design was subsequently developed based on the geomechanical analysis that mitigated the problem with the fourth casing string and led to a significantly less costly well (the heavy black line on the figure).

Because the geomechanical parameters (stress, pore pressure, and strength) are largely out of our control, there are a limited number of things that can be done to minimize geomechanical stability problems. One (as illustrated in Fig. 1.48) is to optimize the locations of casing seats. Another is to optimize mud weight and drilling parameters, minimizing swab and surge while running pipe and maintaining an appropriate pumping rate to keep equivalent circulating density (ECD) low, in situations where it is necessary to maintain a close tolerance. Other options include changing the well trajectory, where that is possible, or at least identifying those trajectories that are least likely to cause drilling problems. An example in which this is particularly valuable is in drilling moderate-reach wells where there is a choice in the depth, length, and inclination of deviated hole sections. It may also be possible by use of appropriate drilling fluids to increase the pressure required to propagate hydraulic fractures, thereby reducing the

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Fig. 1.47—Taken from Wiprut and Zoback, this figure illustrates a case in which frictional constraints combined with observations of wellbore failure, calculated values of $S_v$ and $P_p$, and measured rock strengths provided excellent constraints on the magnitudes of the horizontal stresses. A leakoff test analyzed separately from this analysis confirmed the predicted magnitude of $S_{H_{min}}$. 
leakoff pressure, and recent developments reveal that it may be possible also to increase the leakoff pressure by changing near-wellbore conditions with use of special materials or by heating the well.

To maximize the number of options, geomechanical design constraints should be developed as early as possible in the life of a field, particularly in cases in which development will be carried out from a small number of fixed locations. This way, recovery can be maximized with the smallest number of wells drilled along risky trajectories and the lowest facilities cost.

1.7.1 Predicting Failure in Wells of Any Orientation. Fig. 1.49 shows how wellbore stability in wells of all orientations can be illustrated by a lower hemisphere projection of the likelihood of breakout formation for a single stress state at a single depth. Wells plot on the diagram at locations defined by their orientations. The deviation is represented by radial position (vertical wells plot in the center of the diagram, and horizontal wells plot at the perimeter). The well azimuth is indicated by its circumferential location in degrees clockwise from the top of the diagram; wells deviated to the north (0°) are at the top of the diagram, wells deviated to the east (90°) are on the right side, wells deviated to the south (180°) are at the bottom of the diagram, and wells deviated to the west (270°) are on the left side (Fig. 1.49a).

Fig. 1.49b illustrates the required mud weight to avoid excessive compressive wellbore failure (breakout) as a function of position defined in Fig. 1.49a. Darker gray represents orientations with higher mud weight, and lighter gray represents orientations with lower mud weight, including the strength required for a given degree of failure and mud weight and the amount of failure for a given mud weight and rock strength.
Risk of excessive rock failure around a well can be quantified in a variety of ways—for example, using the normalized radius to which the first episode of brittle failure extends (deep-

Fig. 1.49—The risk of failure as a function of wellbore orientation can be displayed using a lower-hemisphere projection. The construction of this diagram is illustrated in (a). (b) Shows an example plot of the minimum safe mud weight to avoid excessive failure as a function of wellbore orientation as defined in (a) (courtesy GeoMechanics Intl. Inc.).

Risk of excessive rock failure around a well can be quantified in a variety of ways—for example, using the normalized radius to which the first episode of brittle failure extends (deep-
er is worse), or, in the case of analyses using elastoplasticity, the volume of rock that is predicted to reach the yield criterion, the depth of the yielded area, or the onset of a critical plastic strain. In the cases presented here, risk is quantified using the width at the wellbore of the failed zone or breakout. The reason breakout width is preferred is that it is easy to measure using logging data and does not change significantly with time. The same criterion should be used both to determine the magnitudes of the in-situ stresses and to calibrate stability models to improve predictions.

1.7.2 Defining the Mud Window at a Single Depth. Loosely speaking, the mud window can be defined as the range of equivalent densities or pressures that avoid drilling problems. Fig. 1.50 shows how the mud window is defined for a single depth. The lower bound is the mud weight required to prevent excessive wellbore failure as a function of orientation (Fig. 1.50a). Similar figures can be developed to describe the risk of lost circulation, which defines the upper bound of the mud window (Fig. 1.50b). The mud window at a given depth (Fig. 1.50c) is the difference between the maximum mud weight before lost circulation occurs and the minimum mud weight to avoid excessive breakout.

In Fig. 1.50a, the variation in required mud weight to prevent excessive breakout is less than 0.9 lbm/gal. However, the lost circulation pressure (Fig. 1.50b) varies significantly. This is because to generate a lost circulation event, the wellbore pressure must be large enough to do three things: (1) create a fracture at the borehole wall, (2) propagate that fracture through the near-wellbore stress concentration, and (3) extend the fracture against the least principal far-field stress. The far-field stress, of course, is constant, and so the fracture propagation pressure is essentially independent of wellbore orientation. However, the initiation and link-up pressures are strong functions of wellbore orientation. Thus, it can be helpful to choose a wellbore orientation on the basis of maximizing the lost circulation pressure to reach a drilling objective in a low-mud window environment. Notice in this case that the mud window varies from zero for near-horizontal wells drilled to the NW or SE, to 2 lbm/gal for vertical wells, to more than 6 lbm/gal for wells drilled to the NE or SW. In this environment, wells that must be drilled to the NW or SE at this depth should have as small a deviation from vertical as possible.

What is the criterion used to establish the minimum safe mud weight? Clearly, it is one that will minimize the risk of complete hole collapse. But in addition, the volume of cuttings, the inclination of the well, and the position around the well of the breakouts can also influence this value. The cuttings volume and well inclination are important because of hole-cleaning issues. The larger the cuttings volume per unit hole length, the better hole cleaning needs to be. And, because hole cleaning is easier in vertical wells than in deviated wells, vertical wells can accommodate larger amounts of failure. Increases in pumping rate and carrying capacity, or reduced penetration rates, can mitigate the risk associated with excessive cuttings volumes. Because in deviated wells there is considerable pipe contact with the top and bottom of the well, breakouts in these locations are likely to be more problematic than breakouts on the sides of the hole. However, if the well needs to be steered, breakouts on its sides may adversely affect directional control. Because breakout width is a relatively easy measurement that is directly related to cuttings volume, and because breakout depth increases with time, we ordinarily choose the breakout width as the criterion to establish the appropriate minimum mud weight. Because breakouts have been observed that extend more than 100° on each side of a well in vertical wells drilled into some shales, this is an appropriate limit for such wells. Narrower breakouts will become problematic in more brittle rock, so in practice it is best to use a breakout width limit of 90° for breakouts on each side of a vertical well. This limit means that at least half of the wellbore circumference must be intact, a condition that has been referred to as “sufficient to maintain arch support” in sanding analyses. Because hole cleaning is more difficult in deviated wells, the maximum safe breakout width should be reduced as deviation increases.
It is important to remember that it is not necessary to completely avoid breakout formation to drill wells safely. Using such an overly restrictive criterion is not only unnecessary but will
inevitably lead to recommendations for excessively high mud weights in situations in which these are not warranted.

### 1.7.3 Casing Seat Selection.

Analyses illustrated in Figs. 1.49 and 1.50 were carried out at a single depth. However, it is necessary while drilling to maintain stability over the entire open-hole section between casing points. Therefore, analyses of stability must be carried out over that entire depth range. Using the results, the positions of casings can be adjusted to maximize wellbore stability while staying within engineering constraints. While the analysis requires knowing rock properties in detail, it is not necessary to do the calculations using every depth point. This is because although there is considerable variation in rock properties, narrow zones of severely weak rock do not, in practice, cause excessive problems. Furthermore, stresses and pore pressures generally vary slowly with depth and horizontal location. Where wells cross faults, changes in age or lithology, or fluid pressure barriers, abrupt changes in stress and pore pressure are possible. In addition, systematic changes in stress orientation and magnitude often occur adjacent to faults and salt bodies. Provided that the geomechanical model incorporates these effects, it is sufficient to use smoothly varying rock properties. The natural geological variability can be taken into account using statistical methods, as discussed next.

**Fig. 1.51** is an illustration of the impact of geomechanics on casing selection for an offshore well. It shows plots of the equivalent densities of the pore pressure and the leakoff pressure as a function of true vertical depth for a vertical well (for deviated wells, it can be drawn as a function of measured depth). To the right of each figure is shown a casing design diagram. Superimposed on the equivalent mud weight plot are shaded rectangles that represent the limiting mud weights that are both above the minimum required mud weight (in the left plot, the pore pressure, shown in light gray) and below the maximum required mud weight (in all of these plots, the least principal stress, shown in black) at all depths within each casing interval. The upper and lower bounds on the mud weight can be selected from among several different limits. For example, in sections of underpressured sands, the upper limit may be dictated by the pressure above which differential sticking may occur. As shown in very dark gray in the center and right figures, the lower limit could be the collapse pressure computed using geomechanical analysis. And, as discussed in the context of **Fig. 1.50b**, the upper bound to prevent lost circulation can be the pressure required to initiate, to propagate, or to extend a hydraulic fracture.

**Fig. 1.51a** shows a predrill design based on offset experience and the assumption that the pore pressure and the fracture gradient are the upper and lower bounds on the mud window. When geomechanical stability is considered (**Fig. 1.51b**), the results indicate that over a significant portion of the well, the minimum safe mud weight required to avoid excessive breakout development (the collapse pressure) is greater than the pore pressure. One consequence is that the fourth casing section has an extremely narrow mud window. In fact, severe drilling problems developed in this section, necessitating two sidetracks and considerable lost time. On the right is shown a new well design utilizing a geomechanical model to establish safe casing points (**Fig. 1.51c**). This model indicates that it is possible to extend the depths of the second and third casing strings, thereby reducing the required length of the fourth. This not only increases the margin for the fourth casing string, it also makes it possible to reach the reservoir with one less casing than required by the original design.

### 1.7.4 Validating the Geomechanical Model.

It is important when using geomechanical analysis to use prior drilling experience to validate the geomechanical model. This is possible, even when no log data are available for previous wells, by using drilling events such as mud losses, tight spots, places necessitating repeated reaming, and evidence of excessive or unusually large cuttings. If wellbore stability predictions for existing wells are capable of reproducing previous
drilling experience, we can be confident that the geomechanical model is appropriate for use in predicting the stability of planned wells.

**Fig. 1.52** shows an example prediction of the degree of wellbore instability (quantified in terms of breakout width) in a vertical well in deep water. The figure was prepared using the drilling mud program for that well and the geomechanical model developed for the field based on offset experience. The model indicates that while the section above 5,800 ft will be quite stable (no failure is predicted), below that depth, failure will progressively worsen until, at 7,400 ft, it is severe enough to cause considerable drilling problems. Although the model was not able to explain problems encountered in this well above 5,400 ft, it turned out that these problems were not caused by geomechanics because they were mitigated with no change in mud weight, and no evidence of enlargement was found in log data from this interval. In contrast, considerable drilling difficulties were encountered just above 7,800 ft in this well that were detailed in drilling reports, including several packoff and lost-circulation events. These problems required setting casing prematurely at that depth. Single-arm caliper logs subsequently revealed that this section was severely enlarged.
Below the casing point, the mud weight was increased, which reduced hole instability problems in the remaining sections of the well as predicted by the calculations. Nevertheless, there was some evidence for wellbore enlargements in caliper data in the interval below 9,200 ft, even for the higher mud weights used. These sections were those in which the predicted breakout width exceeds 90°, lending support to the validity of the geomechanical model. Subsequently, the model was used to design a number of wells, all of which reached total depth (TD) without incident.

### 1.7.5 Geomechanical Design With Very Little Data

It is not necessary to have a well-constrained stress state to utilize geomechanical design principals. Sometimes, knowing just the stress regime (normal, strike-slip, or reverse), it is possible to estimate relative risk as a function of wellbore deviation and determine the importance of knowing the stress orientation. If geological analysis provides information about stress orientation as well, it is also possible to determine the relative risk as a function of wellbore azimuth.

**Fig. 1.53** shows relative wellbore stability as a function of wellbore orientation at 5,000 ft in normal, strike-slip, and reverse-faulting regimes. In all cases, $S_{H_{\text{max}}}$ is oriented E-W. As can be seen, the required mud weights and their variation with azimuth and deviation are quite different. The lowest mud weights are required in a normal faulting environment. Mud weight increases with deviation when both horizontal stresses are low, and there is only a small sensi-
tivity of mud weight to drilling direction. Required mud weights are higher in the strike-slip regime, and there is a larger variation with drilling direction, especially at higher deviations. Vertical wells require the highest mud weight in this case. To drill in a reverse faulting environment, very high mud weights are necessary regardless of well orientation. The highest mud weights are required for vertical wells and for wells deviated to the North or South (the direction of the minimum horizontal stress), regardless of the amount of deviation. Mud weight decreases with increasing deviation in other directions, and the lowest mud weights are required for wells drilled with high deviations to the east and west. Based on plots similar to Fig. 1.53, it is possible, given only an indication of the stress regime and its orientation (for example, based on the orientations of currently active faults), to define the relative mud weight required for wells drilled at different orientations. If seismic data are available, and the velocity data can be inverted to constrain pore pressure and rock strength, it is possible to make approximate predictions of required mud weights for wells of all orientations.

1.7.6 Handling Uncertainty. In cases in which no wells have yet been drilled in a new exploration area, estimates of required mud weight can have considerable uncertainties. It is, however, possible to quantify those uncertainties and also to learn what measurements are required to provide the maximal improvement in prediction accuracy, using quantitative risk assessment (QRA). QRA analyses can be carried out at a single depth, or over the range of depths between casing seats. Fig. 1.54 is an example of handling uncertainty at a single depth.

In this example, a well is being drilled at a 30° inclination to the north in the strike-slip stress state used to compute Fig. 1.53b. Based on the deterministic recommendation shown in that figure, the minimum mud weight required to drill the well without excessive instability is 14.6 lbm/gal. If at the time of analysis no well had yet been drilled, there would, however, be large uncertainties in the magnitudes of the two horizontal stresses and their orientations. It is also possible that the stress field may be inclined slightly with respect to the vertical. There may also be large uncertainties in the overburden stress, $S_v$, and in the rock strength and pore pressure, even if these had been estimated from seismic data. The parameter values and their uncertainties are shown in Table 1.4. Because QRA is carried out using a Monte Carlo approach, it is possible to allow asymmetrical distributions of the inputs, for example, for the rock strength.

The results of the analysis are shown in Fig. 1.54. This figure plots the cumulative probability of avoiding drilling problems associated with wellbore instability as a function of mud weight. The predicted likelihood of avoiding problems using the mud weight calculated deterministically is only slightly greater than 60%, as shown by the vertical dashed line. To guarantee the well’s success, the mud weight would probably have to be higher.

The sensitivity of the mud weight recommendation to the parameter uncertainties is shown in Fig. 1.55. It can immediately be seen that the largest uncertainty is associated with the poorly constrained value of $C_o$. For higher values, the mud weight required to avoid instabilities is considerably reduced. In addition, the large variation in the magnitude of $S_{H_{max}}$ produces a similarly large uncertainty in the recommended mud weight. The pore-pressure uncertainty results in approximately ± 1 lbm/gal uncertainty. Uncertainties in the magnitude of the minimum stress and in the stress inclination contribute very little. Using these results, it is possible to design a data acquisition and analysis program that achieves the greatest reduction in uncertainty at the minimum cost. In this case, the most cost-effective improvement would result from a better constraint on the rock strength.

Even when rock properties and the stress model are well defined, there can be geological uncertainty based on poorly defined or unknown structure. An example of this is shown in Figs. 1.56 and 1.57 for a horizontal well drilled through hard sandstones containing an unknown distribution of intermittent shaley intervals. In such cases, the uncertainty is not caused by measurement error, but rather by the natural complexity of the structure being drilled. Fig.
Fig. 1.53—Required mud weight to prevent excessive wellbore failure (breakouts) as a function of wellbore orientation at a depth of 5,000 ft, for normal (a), strike-slip (b), and reverse (c) faulting regimes. The gray scales are the same for all three cases.
1.56a shows the distribution of log-derived strengths within this interval obtained from the pilot hole, which was drilled overbalanced. Fig. 1.56b shows the distribution used for the QRA analysis, which has a similar shape. Using this distribution, the QRA analysis of the likelihood of excessive wellbore instability is shown in Fig. 1.57. It is clear from this figure that there is little risk associated with a balanced well. However, a well drilled with a 1 lbm/gal underbalance has only a 66% likelihood of avoiding excessive wellbore failure. Because the company for which the well was drilled was risk-averse, the decision was made not to attempt underbalanced drilling.

Fig. 1.54—This plot shows quantitative risk assessment (QRA) of the cumulative likelihood of avoiding excessive wellbore stability problems (in percent) as a function of mud weight, for the stress state and properties distributions shown in Table 1.4. A well drilled using a deterministic mud weight recommendation (dotted line) has a 67% likelihood to avoid wellbore instability.

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*The expected values are those used to compute Fig. 1.53.*
1.8 Other Models for Wellbore Stability

In many cases, wellbore stability analysis can be carried out with very simple models that are time-independent and relate stress and pore pressure only through the effective stress law. These do not account for the fact that stress changes induce pore pressure changes, and vice versa. Nor do these models account for thermal and chemical effects and their relationships to pore pressure and stress. In this section, we briefly discuss each of these issues and how they affect wellbore stability analysis. We start with a discussion of failure caused by anisotropic rock strength, which is a characteristic of consolidated shales that can cause considerable problems in wells drilled at oblique angles to bedding.

While the examples shown here demonstrate that it is possible to quantify uncertainties in the minimum safe mud weight, it is also possible to quantify uncertainties in the maximum safe mud weight. In that case, the likelihood of success decreases with increasing mud weight, and the two edges of the field defining the most stable mud weights form a possibly skewed bell-shaped curve.

1.8.1 Anisotropic Strength. In many rocks (lithified shales in particular), the elastic properties are anisotropic. In other words, they are a function of the orientation of the applied stress with respect to bedding planes (in general, shales are stiffer along the bedding planes than perpendicular to bedding). At the same time, the rock strength is also anisotropic. In both cases, the anisotropy is caused by a preferred orientation of shale particles that generally becomes more pronounced with compaction.

A well that is drilled perpendicular to shale bedding is generally not affected by bedding-parallel weakness planes. However, when a well is drilled at an oblique angle to bedding, bedding-parallel weakness planes can become very important. Fig. 1.58 shows an acoustic wellbore image of breakouts that occur along oblique bedding planes intersecting a well, demonstrating that this mode of failure does occur. In fact, in this well failure associated with weak bedding caused severe instabilities, necessitating a sidetrack.
Fig. 1.56—(a) Histogram of log-derived $C_o$ for a reservoir interval proposed for underbalanced drilling and openhole completion. (b) Log-normal probability distribution function for $C_o$ consistent with the variation shown in the histogram in (a).\textsuperscript{27} (Reprinted from “Comprehensive Wellbore Stability Analysis Utilizing Quantitative Risk Assessment,” Moos et al., \textit{J. of Petroleum Science and Engineering}, Vol. 38, pages 97–109, © 2003, with permission from Elsevier.)
**Fig. 1.59** is an example plot that shows the required mud weight as a function of wellbore orientation for wells drilled through dipping beds that are highly anisotropic and illustrates the two angles required to define the orientation of a well with respect to bedding. The first angle is the attack angle, which is simply the angle between the well axis and the normal to the bedding plane. The larger the attack angle, the more likely it is that failure will occur because of bedding-parallel weakness planes. The second is the angle between the dip direction and the projection of the well axis onto the bedding plane. While in general it is found that wells drilled updip or downdip are more stable than those drilled along strike, the relationship depends critically on the orientations and magnitudes of the in-situ stresses.

Both of these effects can be seen in the lower right stability plot shown in **Fig. 1.59**, which indicates the mud weight required to maintain stability as a function of wellbore orientation. The bedding normal is shown in this plot as a white dot. The lighter gray shading close to the dot indicates that lower mud weights are required for wells drilled nearly perpendicular to the bedding planes. Darker colors show that high mud weights are required for wells drilled obliquely to bedding. The highest mud weights are required for wells drilled with moderate to high deviations to the ENE. The asymmetry in the plot is a characteristic of the effect of strength anisotropy, and it is caused by the complex interplay between the stress field concentrated around the well and the weak bedding planes. For comparison, the lower left stability plot shows recommended mud weight if there were no weak bedding planes. The difference between the two is the affect of bedding, which requires raising the mud weight if those planes are active. Where bedding planes are not active, similar mud weights are recommended. Notice
that the relative stability of the wells shown on the two plots is quite different when bedding weakness is taken into account from when it is not.

It is very important to realize that, contrary to cases in which wellbore instability is caused by failure of the intact rock, raising the mud weight past a certain point usually exacerbates failure in anisotropic shales. Because failure in these rocks often involves slip along discrete planes, the result is that irregular chunks of rock are often produced, and when cross-cutting fractures are present, the pieces are often spindle-shaped. Raising the mud weight when this type of failure is observed often causes an increase in fluid pressure along the weak planes, reducing their resistance to slip, thereby making failure worse. This problem is often addressed in part by adding fluid-loss-control agents to the drilling mud.

1.8.2 Poroelasticity and Thermoporoelasticity. Poroelasticity theory describes the coupling between pore pressure and stress in rocks. When pore pressure and stress are coupled, fluid diffusion plays an important role, and stability becomes time-dependent. To use the poroelasticity equations developed by Biot to model this process requires knowledge of more rock properties than are required for elastic analyses. These include the elastic moduli, the porosity, the permeability, and a pore pressure-stress coupling term. Even without modeling the problem, however, it is obvious that when a well is overbalanced, fluid diffusion into the rock is likely to cause instability to increase over time. This is because diffusion causes the initial overbalance required to support the wellbore to decrease with time as the near-wellbore pore pressure increases, leading to a decrease in wellbore support and increased failure of the rock. This time-dependent weakening is reduced by development of a mud cake. Thus, it is often observed in wells with strong, brittle shales and weak, high-porosity, high-permeability sands that the

Fig. 1.58—Examples of wellbore breakouts observed in acoustic image data from a well drilled through interbedded massive and laminated sands. Wider breakouts (dark bands) can be seen in the laminated sands above 9,569.5 ft. Where laminations are less frequent, the breakouts are narrower. This pattern indicates wellbore failure in the laminated sands being exacerbated by weak bedding planes.
strong shales break out, whereas the weaker sands appear more intact. An additional reason for the apparently anomalous stability of the sands is discussed briefly in the section on plasticity.

Fig. 1.59—The upper diagram defines two angles that are used to describe the orientation of a well with respect to bedding. The lower figures show the mud weight (in ppg) required to maintain stability as a function of wellbore orientation for a highly anisotropic shale if the weak bedding planes are ignored (on the left) and considered (on the right). Pale grays show that low mud weights are required for wells drilled approximately perpendicular to bedding, whereas darker grays show that higher mud weights are required for wells drilled obliquely to bedding (courtesy GeoMechanics Intl. Inc.).
Fig. 1.60—Poroelastic analysis of failure of a horizontal well drilled through a 10% porosity gas sand with a permeability of 1 μDarcy. The amount of wellbore failure increases with time. Zones of failure shown in the wellbore cross section correspond to the positions of the stars on the lower plot. For example, the light gray line shows the extent of failure 1 minute after the well has been drilled using an equivalent mud weight of 15.5 lbm/gal. The other stars are the times corresponding to the other failure zone outlines.
The effect of fluid diffusion is illustrated in Fig. 1.60. On the top is shown a wellbore cross section. Superimposed on the cross section is a series of contours that define the volume.
of rock in which the stresses exceed the rock strength as a function of time. The heavy gray curves show the boundary of the breakout zones after 1 minute, and the other curves show its shape at 10, 100, 1,000, 10,000, and 100,000 minutes. Although the amount of failure gets larger with time, the width of the failed zone at the wellbore does not change. This is because, in this example, it is assumed that no mudcake forms, and there is perfect communication between the wellbore fluid and the pore fluid. However, away from the well, the amount of failure increases with time. The lower plot shows the total angular coverage of the failure zones as a function of time and mud weight, at a radial distance from the center of the well that is 20% larger than the drilled radius. Although higher mud weights do reduce the amount of failure at short times after drilling, there is a slow but systematic increase in the amount of failure with time, regardless of the mud weight used. The stars in the lower plot show conditions corresponding to each breakout drawn on the well cross section.

Thermal energy transfer obeys the same diffusion law as does the movement of pore fluid. Hence, it is straightforward to model the time-dependent effects of wellbore cooling using the same equations as are used for poroelasticity. This is potentially quite important because cooling a well reduces the circumferential stress and thereby temporarily decreases the likelihood of breakout formation. Simply modeling the pore pressure and temperature independently is not enough, however, because thermal energy transfer occurs both by conduction (heat transfer) and by convection (motion of warm or cold fluids). Thus, a fully-coupled thermoporoelastic theory is required.

Fig. 1.61 shows analysis of the effect of a 30°F reduction in mud temperature for the same parameters used to generate Fig. 1.60. It is clear that failure is much less pronounced when the mud has been cooled. In fact, the analysis indicates that not only can cooling increase the length of time this well remains stable, it may also allow a significant decrease in mud weight. This is because of the contributions of two effects. First, cooling the wellbore reduces the circumferential stress that leads to failure. And second, cooling the fluid reduces the pore pressure, increasing the effective strength of the rock.

1.8.3 Mud/Rock Interactions. From the perspective of wellbore stability, shales are the most problematic lithologies to drill through. Evidence abounds that the shale sections of wells drilled with water-based mud are significantly more rugose than the same sections of similar wells drilled with oil-based mud. The primary reason for these observations is that chemical interactions that occur between shales and water-based drilling muds cause a significant reduction in the effective strength of the shales. Two effects contribute to this problem. The first is osmotic diffusion (the transfer of water from regions of high salinity to regions of low salinity), which causes water in low-salinity mud to diffuse across the membrane formed at the mud/rock interface. The second is chemical diffusion (the transfer of specific ions from regions of high concentration to regions of low concentration). These two effects both change the internal pressure of water in the shale and also affect its strength. Each occurs at a different rate, which in some cases can lead first to weakening and then to strengthening of a wellbore.

When the salinity of the drilling mud water phase is lower than the salinity of the pore fluid, osmotic diffusion causes shales to swell and weaken because of elevated internal pore pressure caused by uptake of water into the shale. Consequently, one solution to shale instabilities is to increase the salinity of the water phase of the mud system, and this works in some cases. However, if the salinity is increased too much, it can cause microfracturing to occur.

In calculating the magnitude of the pressure generated by osmotic diffusion, the parameter that is used to select the appropriate salinity is the water phase activity. Activity (which is explicitly the ratio of the vapor pressure above pure water to the vapor pressure above the solution being tested, and can be measured at the rig with an electrohygrometer) varies from zero to one. Typical water-based muds have activities between 0.8 and 0.9. Typical shales in situ have pore-fluid activities between 0.75 and 0.85, based on extrapolations of laboratory da-
The use of typical muds in typical shales thus causes an increase in the pore pressure within the shale, leading to shale swelling, weakening, and the development of washouts. Mody and Hale published Eq. 1.21 to describe the pore pressure increase owing to a given fluid activity contrast.

$$\Delta P = E_m \left( \frac{RT}{V} \right) \ln \left( \frac{A_p}{A_m} \right). \tag{1.21}$$

If $\Delta P$ is negative, it indicates that water will be drawn into the shale. Here, $R$ is the gas constant; $T$ is absolute temperature, and $V$ is the molar volume of water (liters/mole). Decreasing the mud activity often alleviates shale swelling because $\Delta P$ is positive if $A_p$ (the pore fluid activity) is larger than $A_m$ (the mud activity), and water will be drawn out of the shale for this condition. The parameter $E_m$ is the membrane efficiency, which is a measure of how close to ideal the membrane is. Explicitly, it is the pressure change across an ideal membrane owing to a fluid activity difference across the membrane, divided into the actual pressure difference across the membrane in question. Membrane efficiency is affected both by mud chemistry and by the properties of the shale. In particular, the ionic radius and the pore throat size of the shale appear to play a strong role. Oil-based mud has nearly perfect efficiency. Although water-based mud generally has very low efficiency, some recently developed water-based synthetics have been designed to have high efficiencies approaching those of oil-based mud. Fig. 1.62 shows the relationship between membrane efficiency, mud fluid activity, and degree of failure.

![Fig. 1.62—Plot showing the amount of failure around a wellbore drilled into reactive shale as a function of the water activity of the mud and the membrane efficiency. The shale pore fluid activity is 0.7.](image)
(quantified in terms of the widths of the failed regions) for shale with a nominal pore fluid activity of 0.7. Higher mud activities than the shale pore fluid cause an increase in breakout width, whereas predicted breakout width is less for muds with lower activities. The effect decreases for lower membrane efficiencies.

The model described by Eq. 1.21 is implicitly time-independent, and diffusion is a time-dependent process. Time-dependent models have been developed that predict variations in pore fluid...
pressure due to chemical effects as a function of time and position around the hole. These are explicitly both chemo-elastic and poro-elastic (that is, they account for interactions between the pore pressure and the stress as well as the chemical effects on the pore pressure). The results allow selection of mud weights for specific mud activities, or mud activities for specific mud weights. Fig. 1.63 (top) shows a plot of failure vs. time and mud weight for a shale with a pore-fluid activity of 0.8, for a mud activity of 0.9. As can be seen, failure gets worse over time, and even a mud weight as high as the fracture gradient of 16 lbm/gal maintains hole stability for less than one day. On the other hand, for a mud activity of 0.7 (Fig. 1.63, bottom), the time before failure begins to worsen is extended, and it is possible to select a mud weight below the fracture gradient and yet still provide several days of working time.

### 1.8.4 Wellbore Failure in Plastic Rock

As previously discussed, young, weak rocks that are still undergoing compaction behave plastically. The same can be said of high-porosity reservoir sands. One consequence is that these materials “fail” with only a small reduction in strength. Therefore, wellbore stability modeling can be done more accurately in young rocks using plastic models. But, because the current state of a plastic material is a function of its stress/strain history, fully 3D plastic models require numerical methods. While plastic models are not necessary for extremely brittle rocks, it is not always clear which model is the most appropriate when a rock has intermediate properties.

The simplest way to decide whether it is important to use plasticity is to look at the stress-strain curve of the rock of interest. If it has large strain at failure and it has a detectable yield

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**Table 1.5—Strength Parameters Used for the Analyses Shown in Figs. 1.64 and 1.65**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Onset of Yield</th>
<th>Peak Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohesion, $S_c$</td>
<td>943 psi</td>
<td>1,305 psi</td>
</tr>
<tr>
<td>UCS, $C_o$</td>
<td>3,533 psi</td>
<td>4,890 psi</td>
</tr>
<tr>
<td>Friction angle, $\mu_i$</td>
<td>34°; 0.67</td>
<td>34°; 0.67</td>
</tr>
</tbody>
</table>

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**Fig. 1.64**—Example of modeling failure in a poroelastic-plastic material. Failure is assumed to occur at a plastic strain of $3 \times 10^{-3}$ (3 millistrain). This produces a breakout defined by the white area at the side of the well. The rock properties are shown in Table 1.5.
point, and/or it fails without total loss of strength, plastic models should be considered. That said, no one has yet published a definitive study that demonstrates that an elastic-plastic model is a better predictor of required mud weight for drilling than an equivalent elastic-brittle model. In fact, in cases in which both approaches are used, it is often found that predicted mud weights to avoid excess wellbore instabilities using the two techniques are within 0.1 lbm/gal.

Fig. 1.64 is the output from a strain-hardening, poroelastic-plastic analysis of failure around a balanced well. The stresses, pore pressure, and mud weight are shown in the upper right of the figure. The other parameters, obtained from measurement of the core the properties of which were used in this analysis, are shown in Table 1.5. With a failure model calibrated by laboratory tests, which predicts the onset of failure after 3 plastic millistrain (0.3%), a failed zone is predicted to have a half-angle of approximately 55 degrees, as shown in white on the side of the well. Fig. 1.65 presents a similar analysis, using the same stresses, pore pressure, and mud weight, using a poroelastic-brittle model for failure. In this case, there is a remarkable similarity between the width of failure (the elastic model describes only the initial zone which will deepen with time, as discussed previously) of these two analyses, indicating that it is not necessary to use a plastic model to describe the material.
1.9 Making Decisions in Real Time

In situations in which predrill analysis reveals high risk but has a large uncertainty, it is possible to mitigate that risk by carrying out geomechanical analysis in real time. This can be done but requires acquisition of a variety of data while drilling. Annular pressure measurement using a PWD tool is one key component of real-time stability analysis because knowledge of the hydrostatic and circulating pressures is required to determine the magnitude of kicks, identify borehole ballooning events, and monitor hole cleaning. The measurement can also be used to show where transient pressure events such as surging and breaking the gel strength of the mud exceed fracture pressure, or where swabbing reduces the pressure below the pore or collapse pressure of the wellbore. LWD resistivity, sonic velocity, and bulk density measurements provide information for use in constraining pore pressure and rock strength. Direct pore-pressure measurements while drilling can provide critical data to calibrate pore-pressure predictions in permeable formations. Extended leakoff tests are strongly recommended. Even observations of cuttings shapes and volume can be important to identify the amount and cause of wellbore failure. Because the relationship between rock strength and log data is often poorly known, penetrometer tests and velocity measurements on cuttings are useful both to quantify the

Fig. 1.66—This figure shows examples of time-based (a) and depth-based (b) displays of real-time PWD data, superimposed on event predictions of lost circulation and collapse. They are from two different depth intervals in the same well and can be produced in real time as a well is being drilled.
Fig. 1.66 shows examples of displays of real-time wellbore stability pressure plots. On the left is a display as a function of time, with collapse and lost-circulation pressures predicted using the geomechanical model superimposed on the real-time and recorded data from a PWD tool. As the well was being drilled, a mud-loss episode occurred, which the analysis indicates occurred because the downhole ECD exceeded the fracture gradient. This indicated that the model was fairly accurate at that depth. Deeper in the hole, data recorded while drilling indicated that an adjustment needed to be made to the geomechanical model. Fig. 1.66b shows a plot of predicted mud weights as a function of depth on which the real-time PWD pressure data have been superimposed. In this case, the ECD is high enough to avoid drilling problems, but it appears that the static mud weight is very close to the minimum required to avoid excessive instability. Based on this observation, high cuttings volumes should be expected, and one recommendation would be to take extra care to avoid high running speeds and accelerations that might swab the hole.

A key component of real-time analysis is to provide an understanding of the origin of problems to make the right adjustment to drilling parameters to compensate. For example, when drilling through shales with inhibitive (chemically nonreactive) mud, fluid leakage may change the mud characteristics over time. When would an adjustment to the mud properties be required? A comparison of stability risk vs. mud chemistry could help make that decision. Or what happens if the rock strength suddenly decreases due to crossing from relatively strong shale into one that is much weaker? To handle this case, a crossplot of breakout width as a function of $C_o$ and mud weight can be used to determine the amount of mud weight adjustment required.
Figs. 1.67 and 1.68 show a hypothetical case of using geomechanical decision tools to help evaluate various drilling options when crossing from a strong shale into one that is substantially weaker and highly reactive. In Fig. 1.67, it is apparent for this hypothetical case that to maintain the same degree of stability if $C_o$ decreases from 4,000 psi to 2,000 psi, it is necessary to increase the mud weight to a value that exceeds the fracture gradient. This makes it impossible to continue drilling, and the options are either to stop and set casing or to investigate other possibilities.

Plots such as Fig. 1.68 that shows breakout width as a function of mud weight and mud activity, can reveal whether it is possible to compensate for changes in strength by changing mud chemistry. Since breakout width in this particular shale is a strong function of mud activity, to compensate for the change in strength without changing mud weight it is only necessary to decrease the mud activity from its current value of 0.8 to a value of 0.6. This would achieve the desired result with no change in mud weight. It is also possible, of course, to maintain safe drilling conditions using a combination of a mud activity reduction and a mud weight increase.

The information presented in this chapter is only an introduction to the theory and application of geomechanics to drilling. It is a very young field, and rapid advances are being made. One example of this is a new approach that has been developed to model the near-wellbore behavior of fractured rock. Another is the application of uncertainty analyses to pore-pressure prediction. At the same time, new engineering techniques are being developed that provide solutions to geomechanical problems. For example, the same analysis as presented in Fig. 1.51 can be carried out for dual-gradient drilling, to quantify the potential of the technique to extend...
casing seats to greater depths in deep water. It is important to realize that the same geomechanical models that help improve drilling efficiency can be shared among drillers, geologists, and reservoir engineers and used to help improve operations throughout the life of a field. In fact, the best use of geomechanics is to develop initial models as early as possible and to use these models in every phase of field development, updating and refining the models as new information is obtained.

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Nomenclature
\( a, b, c, d, e \) = constants used in Eq. 1.15 and tabulated in Table 1.4
\( A_m \) = mud activity, ratio
\( A_p \) = pore fluid activity, ratio
\( B \) = Skempton’s coefficient, unitless
\( C_{\text{eff}} \) = effective strength, MPa, psi
\( C_o \) = unconfined compressive strength, MPa, psi
\( D_c \) = drilling exponent
\( e \) = base of a natural logarithm, \( e = 2.718281828 \ldots \)
\( E \) = Young’s modulus, GPa
\( E_m \) = membrane efficiency, ratio
\( f \) = acoustic formation factor, used in Eq.1.19
\( G \) = acceleration of gravity, m/s²
\( h \) = vertical height of a thin planar reservoir, length
\( K \) = Bulk modulus, GPa
\( K_{\text{dry}} \) = Bulk modulus of the dry frame of a porous rock, GPa
\( K_{\text{grain}} \) = Bulk modulus of the grains that make up the rock, GPa
\( L \) = horizontal length of a thin planar reservoir, length
\( l_k \) = length in the \( k \) direction
\( l_l \) = length in the \( l \) direction
\( M_{ijkl} \) = Component of the modulus tensor that relates the \( ij \) component of the stress tensor to the \( kl \) component of the strain tensor, MPa, psi
\( P_a \) = Pore pressure at depth \( a \)
\( P_c \) = confining pressure, MPa, psi
\( P_{\text{hyd}} \) = hydrostatic pressure, MPa, psi, lbm/gal
\( P_m \) = pressure of mud in a well, MPa, psi, lbm/gal
\[ P_p = \text{pore pressure, MPa, psi, lbm/gal} \]
\[ P_z = \text{Pore pressure at depth } z, \text{ MPa, psi, lbm/gal} \]
\[ R = \text{gas constant, J/mol/°K} \]
\[ R_{\text{log}} = \text{measured value of resistivity, ohm-m} \]
\[ R_n = \text{normal value of resistivity, ohm-m} \]
\[ S_1 = \text{greatest principal stress, MPa, psi} \]
\[ S_2 = \text{intermediate principal stress, MPa, psi} \]
\[ S_3 = \text{least principal stress, MPa, psi} \]
\[ S_{11} = \text{stress component acting normal to a plane perpendicular to the } x_1\text{-axis, MPa, psi} \]
\[ S_{12} = \text{stress component acting in the } x_2\text{-direction along a plane perpendicular to the } x_1\text{-axis, MPa, psi} \]
\[ S = \text{total stress, MPa, psi} \]
\[ S_a = \text{stress at depth } a, \text{ MPa, psi, lbm/gal} \]
\[ S_{\text{Hmin}} = \text{least horizontal stress, MPa, psi, lbm/gal} \]
\[ S_{\text{Hmax}} = \text{greatest horizontal stress, MPa, psi, lbm/gal} \]
\[ S_{ij} = \text{component of the stress tensor acting in the } x_j\text{ direction on a plane perpendicular to } x_i, \text{ MPa, psi} \]
\[ S_{ji} = \text{component of the stress tensor acting in the } x_i\text{ direction on a plane perpendicular to } x_j, \text{ MPa, psi} \]
\[ S_n = \text{normal stress acting on a plane, MPa, psi} \]
\[ S_o = \text{cohesion, MPa, psi} \]
\[ S_z = \text{axial stress along a wellbore, MPa, psi} \]
\[ T = \text{absolute temperature, °K} \]
\[ T_o = \text{tensile strength, MPa, psi} \]
\[ V = \text{molar volume of water, liters/mole} \]
\[ V_p = \text{compressional-wave velocity, km/s} \]
\[ V_s = \text{shear-wave velocity, km/s} \]
\[ x_1, x_2, x_3 = \text{Cartesian coordinate system} \]
\[ x, y, z = \text{Cartesian coordinate system} \]
\[ Z_o = \text{depth, ft, m} \]
\[ \alpha = \text{Biot poroelastic coefficient} \]
\[ \beta = \text{term used in equation 1.20} \]
\[ \delta_{ij} = \text{Kronecker delta (} \delta_{ij} = 1, \text{ if } i = j; \delta_{ij} = 0 \text{ otherwise)} \]
\[ \Delta = \text{operator indicating a change in a parameter (} \Delta P_p \text{ is change in } P_p \) \]
\[ \Delta P = \text{difference between the pressure of fluid in a well and the pore pressure} \]
\[ \Delta t_{\text{ma}} = \text{matrix transit time, } \mu_s/\text{ft} \]
\[ \Delta T = \text{temperature difference between the fluid in a well and the adjacent rock} \]
\[ \Delta T_{\text{log}} = \text{measured value of sonic transit-time at a given depth, } \mu_s/\text{ft} \]
\[ \Delta t_n = \text{normal value of sonic transit-time at a given depth, } \mu_s/\text{ft} \]
\[ c_{kl} = \text{component of strain acting in the } l\text{ direction per unit length in the } k\text{ direction} \]
\[ \theta = \text{angle around the wellbore measured from the } S_{\text{Hmax}}\text{ direction, degrees} \]
\[ \theta_b = \text{angle between the } S_{\text{Hmax}}\text{ direction and the edge of a breakout, degrees} \]
\[ \mu = \text{coefficient of sliding friction on a pre-existing weak plane, where } \mu = \tan \Phi \]
\[ \mu_i = \text{coefficient of internal friction, where } \mu_i = \tan \Phi \]
\[ \rho = \text{density, } \text{gm/cm}^3 \]
\[ \rho_b = \text{bulk density, } \text{gm/cm}^3 \]
\[ \rho_{\text{log}} = \text{measured value of density, } \text{gm/cm}^3 \]
\[ \rho_n = \text{normal value of density, } \text{gm/cm}^3 \]
\[ \nu = \text{Poisson's ratio} \]
\[ \sigma = \text{Terzaghi effective stress, MPa, psi} \]
\[ \sigma_1, \sigma_2, \sigma_3 = \text{maximum, intermediate, and least effective stresses, MPa, psi} \]
\[ \sigma_{\text{Hmin}} = \text{minimum horizontal effective stress, MPa, psi, lbm/gal} \]
\[ \sigma_{\text{Hmax}} = \text{maximum horizontal effective stress, MPa, psi, lbm/gal} \]
\[ \sigma_{ij} = \text{effective stress acting in the } i \text{ direction on a plane perpendicular to the } j \text{ direction, MPa, psi} \]
\[ \sigma_n = \text{effective stress acting normal to a plane, MPa, psi} \]
\[ \sigma_o = \text{mean effective stress, MPa, psi} \]
\[ \sigma_{rr} = \text{effective normal stress acting in the radial direction, MPa, psi} \]
\[ \sigma_v = \text{effective normal stress acting in a vertical direction, MPa, psi} \]
\[ \sigma_{zz} = \text{effective normal stress acting on a plane perpendicular to the } z \text{ direction, MPa, psi} \]
\[ \sigma^{\Delta T} = \text{thermal stress induced by the cooling of the wellbore by } \Delta T, \text{ MPa, psi} \]
\[ \sigma_{\theta\theta} = \text{the effective hoop stress, MPa, psi} \]
\[ \tau = \text{shear stress, MPa, psi} \]
\[ \Phi = \text{porosity} \]
\[ \Phi_b = \text{breakout width, degrees} \]

**Subscripts**

- \( i \) = index
- \( j \) = index

**Superscripts**

- \( \beta \) = coefficient multiplying the effective vertical stress in Athy’s relationship, Eq. 1.19
- \( \sigma_v \) = effective vertical stress in Athy’s relationship, Eq. 1.19

**References**

General References


### SI Metric Conversion Factors

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion Factor</th>
<th>SI Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>bbl</td>
<td>× 1.589873</td>
<td>m³</td>
</tr>
<tr>
<td>ft</td>
<td>× 3.048*</td>
<td>m</td>
</tr>
<tr>
<td>in.</td>
<td>× 2.54*</td>
<td>cm</td>
</tr>
<tr>
<td>lbf</td>
<td>× 4.448222</td>
<td>N</td>
</tr>
<tr>
<td>ibm</td>
<td>× 4.535924</td>
<td>kg</td>
</tr>
<tr>
<td>psi</td>
<td>× 6.894757</td>
<td>kPa</td>
</tr>
</tbody>
</table>

*Conversion factor is exact.*